Linking Single Period Attribution Results

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The author introduces a new multiperiod attribution model, wherein the linking of time periods generates output with no residual—a property that would appeal to those who examine the past performance of active fund managers.

Within fund management, attribution analysis identifies the sources of an active manager's past performance over some time period. Past performance here is relative: A manager's portfolio return is compared with that of a predefined benchmark. Common sources, or attribution effects, of past relative performance include security selection (how well the manager chose mispriced individual securities) and asset allocation (how well the manager invested across such broad security categories as equities and bonds).

Attribution effects can be linked across time periods. For example, five daily effects can be linked, forming a cumulative weekly effect, four of which can be linked, forming a cumulative monthly effect, and so forth. This linking can be either additive, whereby attribution effects in each time period are added together, or geometric, whereby attribution effects in each time period are multiplied together.

Because cumulative returns for both the active manager's portfolio and the portfolio's benchmark stem from multiplying (or compounding) returns across time, the geometric linking of attribution effects best captures the spirit of performance analysis. Geometric linking, however, typically generates a residual, which must somehow be explained. Additive linking sums wholly across time and thus, by definition, does not leave a residual.

The author asserts that audiences intuitively understand attribution effects that add wholly across time, and he proposes a mathematical solution that allows geometric linking but *without* a residual. Proclaiming a dearth of academic literature on multiperiod attribution

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models, the author introduces mathematical procedures, called the Frongello linking algorithm (FLA), that address multiperiod linking. The author demonstrates that his eponymous algorithm generates no residual in a mock two-asset portfolio across three identical time periods for two attribution effects (asset allocation and security selection).

The FLA is a one-line equation with four inputs—the unadjusted attribution effect in the current period, the return of the active portfolio in the previous period, the return of the benchmark in the current period, and the value of the FLA in the previous period. In essence, FLA scales prior-period attribution effects separately from those in the current period so that cumulative attribution effects are unaffected by their predecessors over time.

The author compares the results of his FLA with the results of algorithms by Cariño (*Journal of Performance Measurement*, 1999) and Menchero (*Journal of Performance Measurement,* 2000). When the three single periods have identical relative returns and attribution effects, the three models produce the same results. But when period returns and attribution effects vary, the three models produce different results, with the Cariño and Menchero algorithms generating similar output. (The author notes that the Cariño algorithm emphasizes periods with below-average returns, whereas the Menchero algorithm evenly recognizes all periods.) Still, all three methods, irrespective of return assumptions, do not generate residuals.

Cariño provided three standards of judgment for linking algorithms: The algorithm must exhibit generality across every kind of attribution effect, familiarity with single-period results, and no residual, where every attribution effect is fully accounted. The author then adds three standards of his own: sincerity, meaning no mathematical "fudging"; intuitive methodology, meaning that the algorithm can be understood by those with basic quantitative skills; and order dependence, wherein the sequence of relative returns affects the calculations.

The author reproves the two competing algorithms because they ignore order dependence, failing his final standard of judgment. The author claims that the FLA is the best approach to multiperiod attribution because it meets all six standards of judgment, particularly his newly introduced standards.

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