'01 Games Dart Outs Explained

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I'm not a world champion of darts, so what qualifies me to cast my vote into these forever ongoing debates? First, I'm extremely passionate about darts. Second, I have a unique set of skills including mathematics, statistics, and programming that can lend somewhat of an unbiased scientific approach to a conversation this is usually dominated by highly personal and emotionally grounded arguments.   I have used these skills in the financial world to solve some complex problems and the algorithms that I've published have received industry wide acceptance and have been incorporated into investment systems around the globe. Addressing the optimal Outs for '01 games should be an easy and fun exercise for me.

My most current out chart can be downloaded here:

http://www.frongello.com/support/Works/FrongelloDartOutChart.pdf

As of 10/3/2018 it looks like this.

Best '01 target = T20 for even, T19 for odd. Exceptions below.

Italics = 2 Darts   <u>Underline</u> = Pressure or Novice

Warning: This is not a traditional outchart.

135	B	74	T14 *T16*	49	17 <u>9</u>
132	B	72	T16	48	16 <u>16/8</u>
128	T18	71	T13 *T19*	47	15 <u>7/19</u>
127	T20	70	T10 *T20*	46	14 <u>6/10</u>
126	T19	69	T11 *T19*	45	13 <u>19/7</u>
125	B	68	T12 *T18*	44	12 <u>8/16</u>
123	B <u>T19</u>	67	T9 *T17*	43	11 <u>3/19</u>
122	B <u>T18</u>	66	T14 *T16*	42	10 <u>10/6</u>
121	B <u>T17</u>	65	B	41	9 <u>17/3</u>
117	T20	64	T16 *T14*	39	7 <u>7/19</u>
104	T16 *T18*	63	B	37	5 <u>17/3</u>
101	T20 *T17*	62	T10 *T12*	35	3 <u>3/19</u>
94	T18	61	B	33	1 <u>17/3</u>
91	T17	60	20	31	15 <u>7/19</u>
90	T20 *T18*	59	19	29	13 <u>17/3</u>
88	T16	58	18	27	11 <u>3/19</u>
87	T17	57	17	25	9 <u>17/3</u>
86	T18	56	16	23	7 <u>7/19</u>
85	B *T15*	55	15	21	5 <u>17/3</u>
84	B *T16*	54	14	19	3 <u>3/17</u>
83	B *T17*	53	13	17	1 <u>5</u>
82	B *T14*	52	20 <u>12</u>	15	7 <u>3</u>
81	B *T19*	51	19 <u>11</u>	13	5 <u>1</u>
78	T18	50	18 <u>10</u>	<13	Leave 2^x
75	T17				

This outchart and discussion commentary at www.frongello.com

You'll notice a couple things about my out chart:

1) It's not really an out chart in the traditional sense where you have the recommended triple and double to finish. **I present only what the very next target should be given a particular situation and remaining score.** When I started playing darts there were a lot of misses. So more often than not, it didn't really matter what came after the triple because the triple didn't get hit that often. Early in my dart journey, even the target single was missed. So for a long time the recommended double indicated after the triple was almost never used. After I've been playing for a while I decided my out chart looked less cluttered without the double and I was good enough at math that including the doubles wasn't necessary. My mindset became "Given what's left…what's my next target?" That's all the really matters. Let's cross the bridge of hitting the double or more likely identifying the next appropriate target after the fallout of where the first dart ends up when we get there. There were far more likely places the first dart would end up besides the triple and I was not about to list all the permutations on my sheet. This would incorporate redundancies in my sheet making it much longer than necessary and also opening up the danger of "internal inconsistencies" in my sheet. More on this next.

2) **There are no "internal inconsistencies" in my sheet**. Almost every out chart I've ever studied has "internal inconsistencies". I will explain what I mean through an example. One of my good buddies that I play with every Friday night is one of the best players I know. He's been working on his out chart for years. But recently I noticed the following: In his out chart he recommends for 145 - T20 T19 D14. This is a perfectly suitable out. He also recommends 85 – T15 D20. Another perfectly suitable out. Each of these individually is just fine. But if these appear on the same sheet it begs the question…..what does this person prefer for 85? T19 D14 or T15 D20? Anytime an out chart labels all the follow on shots, you will have this risk. The first thing I do when I look at an outchart is look for these inconsistencies. In my opinion, the better approach is to recommend to aim for T20 with 145 …..and then if necessary look for the recommendation for 85. This is the technique I use and it completely avoids the internal

consistency problem. If at some point in the future my views on 85 change and I prefer a different out, I simply change my recommended target for 85. I don't have to go through the rest of my out chart and look for any sequence that might also have used that earlier recommendation for 85 and change the sequence there as well.

3) **I start my out chart at 135.** 90% of out charts I've seen look like this for outs above 135.

170	T 20	T 20	DB		149	T 20	T 19	D 16
167	T 20	T 19	DB		148	T 20	T 20	D 14
164	T 20	T 18	DB		147	T 20	T 17	D 18
161	T 20	T 17	DB		146	T 20	T 18	D 16
160	T 20	T 20	D 20		145	T 20	T 19	D 14
					144	T 20	T 20	D 12
158	T 20	T 20	D 19					
157	T 20	T 19	D 20		143	T 20	T 17	D 16
156	T 20	T 20	D 18		142	T 20	DB	D 16
155	T 20	T 19	D 19		141	T 19	T 16	D 18
154	T 20	T 18	D 20		140	T 20	T 20	D 10
					139	T 20	T 13	D 20
153	T 20	T 19	D 18					
152	T 20	T 20	D 16		138	T 20	T 18	D 12
151	T 20	T 17	D 20		137	T 20	T 15	D 16
150	T 20	T 18	D 18		136	T 20	T 20	D 8

If you ask me, this portion of the chart adds zero benefit over me saying "Over 135, aim for T20". I'm saying the same exact thing, just saying it much more efficiently. Also, let's also say for arguments sake you've never played darts before and you don't know that 170's out shot is T20 T20 DB. Even if you have Phil Taylor natural ability and it's the one time in 10 that you are left with 170 and manage to hit the T20 T20......it is 3rd grade math

to figure out that you have 50 left with one dart. You really don't need a chart to tell you to shoot at DB.

Note: You may have noticed that this out chart has a T19 T16 D18 recommendation for 141. T20 T19 D12 is just as difficult and in fact better. 36 halves to 18 and then 9. 24 halves to 12 and then 6 and then 3. So that one exception is crap. My suggestion is better.

4) **I've done my best to incorporate all the variables that matter in deciding the best target.** These are:
   a. How skilled you are?
   b. How many darts are in hand?
   c. Whether you have time or are under pressure?
   Most out charts don't address these. Sometimes you'll see a good one that gives the 2 dart option but I've NEVER seen one that takes into account (or attempts to in my case) your approximate skill level and whether you are under pressure or not.

5) **My out chart is quite concise.** I have identified the variables above that influence the result of the ideal target for a current out. There are three variables that could be thought of as discreet or binary (Skilled vs. Unskilled, 3 vs 2 darts, and With Time vs Under Pressure) so it is very logical and reasonable to assume that there could possibly be 8 different answers to the best target given the state of each of these variables. What I've found is that often times this is the case but usually there are one or two outs that work very well in each scenario. I have made judgement calls in these cases. For example, for 63 my math indicates that T13 is very slightly negligibly marginally better than B in most scenarios involving 3 darts (yes that's right 3 darts) and B is magnitudes better with scenarios involving 2 darts (Yes that's right again….B with 2 darts….we'll get to this later), so here I recommend B in all circumstances.   Adding a T13 recommendation in addition to Bull when 3 darts are involved does not add enough benefit to warrant complicating the sheet.   Another good example, is that often the best option for the novice shooter is also the best option for the skilled shooter under pressure. And in some situations a small judgement call is

used to pick something that although not ***mathematically equivalently*** optimal is in most circumstances ***practically equivalent***. This is why on my sheet although I have modeled skill and time/pressure separately and they sometimes produce different optimal targets…..I've lumped them together and picked the target the best works for both.

6) **Some outs are omitted.** As mentioned earlier, for all outs over 135 just start with T20. No need to include them. Below 135, the out chart is only exceptions to the following rule: ***Start with T19 for odd outs and T20 for even outs.*** I didn't want to include a couple dozen extra lines if this simple rule of thumb suffices.

7) **B represents the entire bull area.** Although I've modeled in probabilities for both….I assume when you are shooting for the bull you are actually aiming for the bull as a whole. I am not at the level where I can differentiate going for a single bull vs a double bull. I aim for the bull in total and deal with what I get. The assumption is roughly 1 in 6 will be a double bull, so I aim at the center of the bull and assume I'm hitting a single. Even if you are aiming for double bull vs. single bull I doubt many players out there will be in a much different spot than me in terms of outcome. Likely 1 in 6 of hits within the bull's eye will be a double bull whether you are a novice or a skilled shooter.   But more power to you if you can easily land darts within the bull and double bull discriminately. If this is the case you probably don't need to read any further. Anyway, from here on out for the recommendation I am just going to say B. But by all means aim for the center of DB.

8) One thing you won't notice is that on other rare occasions I vetoed mathematically marginally better solutions that are too controversial. For example, instead of recommending 6/10 wire with 3 darts in hand and 14 left to go and having to deal with hundreds of people that haven't looked at the math or invested as much time and effort as me into studying this stuff and listening to them go off on me saying "WTF!?!!??!?!?! 14 is D7 ….. what is wrong with you going at 6/10 wire?". To avoid the drama and because 6\10 is not really that much better than D7…..I acquiesce in the spirit of

peace and simplicity and harmony to shoot at the D7 instead. Don't even get me started on shooting at the wire between 14 and D14 with 60 left to go……20 D20 is just fine and honestly equivalent enough not to warrant having a fight over.

My approach and the frustration that motivated my efforts is basically driven by almost every conversation I've ever had with dart players in bars. When you ask any dart player why they prefer a certain out they will go through the 3 (maybe 4 if you are lucky) permutations that are awesome enough to them to convince them to go that way. They don't mean any harm but I am always left wanting more. It's a personality flaw with mathematical types I suppose. I am always left wondering to myself **"Are those the only permutations of events you've considered?"** and **"What are the probabilities you are associating with each of those outcomes?"** For the record, I'm not emotionally attached to any particular out. Unlike many others in bars that react like you've insulted their children when you question the out they are using, I always welcome the debate. However, I have never met anyone that addresses the conversation with as much rigor as I have or am prepared to.   I can tell you the answer to my wonderings; the permutations of events that I've modeled and painstakingly considered are in excess of 20,000 paths when taking three turns at the line. And I've meticulously computed each of these paths for every out. These have been explicitly modeled. Every variation of hitting triple, missing left into double, missing double, hitting single bull, hitting single, hitting double left, blah blah blah, etc. ***Every possible variation of outcome with 9 darts I have modeled explicitly.*** I've also thrown thousands of darts at a board and meticulously taken notes to compute my average hit percentages on targets and neighboring regions based on the target I'm aiming for. I don't sleep well and these questions have relentlessly haunted me….so I did everything I am telling you I did. Every single outcome path has been matched to these measured probabilities and the statistics have been computed to present the aggregate joint probability of each outcome along each path. This was not easy to do. But it's done! The mathematics are specific to my throw and hit percentages. However, I do not believe the variance of my throw has any particular bias or distribution profile unlike any other darter's. I believe the center of my expected target result is exactly where I am aiming (I've become a pretty good shooter and have built a local reputation as being an accurate

shooter) and the variance characteristics of my shot outcome is not unlike most other's. Variance is randomly distributed around the target point. To model the hypothetical unskilled player I just degraded my variance enough to the point that only 2 out of 3 darts hit the single that is aimed at. To model pressure vs time, I look at the closing percentage over 3 darts and compute a rank and look at the closing percentage over 9 darts and compute the rank. Similarly, I look at the best closing results starting with 2 and 3 darts. So yes this is 8 simulations over all outs along all those 20,000 or so paths. The output is an unemotional and unbiased reflection of the inputs. Again, I am not emotionally attached to any out. This is simply the output of my modeling along with some small aforementioned liberties taken in the attempt to keep the out chart concise. Feel free to email me if you want to discuss the approach further. Let's get into the outs.

In my presentation I am going to start from the bottom and work my way up. The reasoning is quite simple. I want to build up solutions for lower outs and use these as building blocks in later more complicated outs.

IMPORTANT: In my outchart and this write up, targets in italics are the 2 dart option if there is a clearly better route with 2 darts vs the 3 dart recommendation. Most of the time there won't be a 2 dart option presented because there is no 2 dart option materially better in closing percentage than the 3 dart recommendation. Also, in underline will be the less skilled or under pressure option. I've lumped these together because they often recommend the same target and it makes sense if you think about the reason why. Both the skilled shooter under pressure and the unskilled shooter need to optimize their chance of closing soon with some freedom for mistakes. Wire shots are denoted with a "/" between the adjacent numbers. The number on the left is the preferred result and the number on the right is the safety. Throughout this discussion "practically optimal" target will mean there is no negligible difference in closing probability from the mathematically optimal target – Often chosen because it's more popular or more exciting or at a more common target or less controversial or easier to remember or is mathematically optimal in some situations but practically optimal in others. In other words, we won't rock the boat unless there is a significant benefit to do so.

<13 – Leave 2^X   For any number less than thirteen that is not even, shoot for the number that leaves you the largest power of 2. 11, shoot for 3 to leave 8. 9, shoot at 1 to leave 8. 7, shoot at 3 to leave 4. 5, shoot at 1 to leave 4. 3, shoot at 1 to leave 2.

13 – 5 <u>1</u>   5 is the best option here. 5 leaves 8 which halves nicely. 1 is the very best target for weaker shooters and skilled shooters under pressure. 1 avoids bust risk on the T5 and 12 halves nicely as well.

15 – 7 <u>3</u>   7 is the best option here. 7 leaves 8 which halves nicely. 3 is the very best target for weaker shooters and skilled shooters under pressure. 3 avoids bust risk on T7 and 12 halves nicely as well.

Often you hear arguments for 3,11,7, and 13 because these offer varying degrees of side bust risk. What I've found is that although 7 is roughly 3% inferior to 3 and 11 for skilled shooters, it is as much as 15% better with time and only a slight benefit to using anything other than 7 and only in a few situations. So for me 7 is the easy choice to stick with and commit to memory.

17 – 1 <u>5</u>   1 leaves 16 which halves nicely. 5 is the practically optimal target for the weaker shooter. 9, 13, and 1 all provide negligibly better percentages with time compared to 5, but 5 is superior under pressure.   The difference in mathematics is not worth remembering these other outs. Skilled shooters should almost universally shoot at 1 unless under pressure when they can opt for 5 as well. Something for the skilled shooter to keep in mind…..shooting at the 5 is better under pressure but comes at the cost of not being as good as 1 with time. 5 also has less side risk as a miss into 12 keeps you in the game.

13, 9, and 5 are all slightly better than 1 for the skilled shooter with pressure. However, you sacrifice a disproportionate amount of success over time going for anything but 1. For example, the skilled shooter is 4.2% more likely to close in 3 darts starting with 5, unfortunately the skilled shooter is 11.0% less likely to be successful over 9 darts starting with 5. This is a tradeoff that is not worth it. Rather than complicate the sheet, I just always go 1.

19 – 3 3/17   3 is the very best target for skilled shooters under all situations except with 2 darts and pressure. In this situation skilled shooter should go for 3/17. However, this is not the optimal play for the skilled shooter under pressure with 3 darts. Best play is still 3. For the less skilled shooter, 3/17 is practically optimal in all situations. So for 19, we are basically talking about the same target…. 3 but if you are going to miss….miss into 17. Based on your skill and situation make a judgement on how close to the right you want to aim toward the wire. Only a miss into 17 keeps you in the game.   19 on the other side busts.

15, 11, and 7 are comparable for the skilled player under pressure however these options sacrifice a significant percentage of success over time. 3 is by far the best overall.

21 – 5 17/3   5 is practically optimal for all situations for the skilled shooter. With pressure 17/3 is better for the skilled shooter but negligibly so. 17/3 is practically optimal for the less skilled shooter in all situations.

For the skilled shooter, 17, 1, 13, 9 are all comparable under pressure. However, these options sacrifice opportunities over time considerably over 5.

23 – 7 7/19   7 is practically optimal for all situations for the skilled shooter. With pressure 7/19 is better for the skilled shooter but negligibly so. 7/19 is practically optimal for the less skilled shooter in all situations.

19, 3, and 11(less so) are negligibly better than 7 with pressure however these routes sacrifice significant success probability over time.

25 – 9 17/3   9 is practically optimal for all situations for the skilled shooter. With pressure and 2 darts in hand, 17/3 is better for the skilled shooter. 17/3 is practically optimal for the less skilled shooter in all situations.

1, 5, 17, 13 all offer a negligible benefit to the skilled shooter under pressure over 9, however, these options sacrifice a significant amount of success over time.

27 – 11 3/19   11 is practically optimal for all situations for the skilled shooter. 3/19 is practically optimal for the less skilled shooter and for the skilled shooter under pressure.

3,19,7 all offer a negligible benefit over 11 for the skilled shooter at the expense of significant sacrifice of success percentage over time.   It's not worth it.

29 – 13 17/3   13 is practically optimal for all situations for the skilled shooter. 17/3 is practically optimal for the less skilled shooter and for the skilled shooter under pressure with 2 darts.   Even under pressure, the skilled shooter can stay with 13 with 3 darts in hand. 13 actually also works for the less skilled shooter with time but negligibly so over 17/3.   Keep in mind though that we are really splitting hairs here and following as presented is just fine.

5,9,17 all offer negligible benefit over 13 for skilled shooter with pressure, however these routes sacrifice significant closing probability over time.

31 – 15 7/19   15 is practically optimal for the skilled shooters in all situations except when there is pressure. 7/19 is best for skilled shooters under pressure and unskilled shooters.

19,11,7 offer negligible better success for the skilled shooter over time but at the cost of a disproportionate sacrifice in success under pressure.

33 – 1 17/3   1 is practically optimal for skilled shooters in all situations except when there is pressure with 2 darts. 17/3 is best for skilled shooters under pressure and unskilled shooters. Unskilled shooters with time can opt for 1 instead of 17/3. With time 1 is slightly better for unskilled shooters than 17/3.

17, 9, 13 are practically equivalent to 1 for the skilled shooter under pressure. Each of these is inferior to 1 overall. If you had to pick one though…..17 is second best.

35 – 3 3/19 3 for the skilled shooter is always optimal. For the less skilled shooter, 3 is a good option but 3/19 is relatively equivalent and highlighted to illustrate that 19 is the better miss to leave 16.

37 – 5 17/3 5 for the skilled shooter and less skilled shooters with plenty of time. For skilled shooters under pressure with 2 darts and less skilled players under pressure 17/3 is best.

17,13,1 offer comparable options for the skilled shooter under pressure but at a huge sacrifice of success over time. I have heard arguments for 7, 19, 3….but these don't make sense to me at all. Far better options are available. 5 is best.

39 – 7 7/19 7 is for skilled shooters and less skilled players with plenty of time. For all shooters under pressure 7/19 is the best target.

41 – 9 17/3 9 is for skilled shooters and less skilled players with plenty of time. For skilled shooters with two darts and pressure and unskilled shooters under pressure 17/3 is the best target.

42 – 10 10/6 10 is for skilled shooters with time. All other situations call for 10/6.

43 – 11 3/19 11 is for skilled shooters with time. All other situations call for 3/19.

44 – 12 8/16 12 is for skilled shooters and unskilled shooters with plenty of time. Skilled shooters with pressure and 2 darts in hand and all other situations call for 8/16. An almost equivalent pressure option is the 4/18 split. This gives the common D20 outshot but if 18 is hit with first dart, you are left with D13 which doesn't leave a shot at out if a single 13 is hit in first outshot attempt. 8/16, although less common doubles, offers two chances at an out because both numbers leave outs that halve twice.

45 – 13 <u>19/7</u> 13 is for skilled shooters in any situation. 13 also works well for less skilled shooters but in situations with pressure they are better going at 19/7.

5 and 9 offer similar success rates for skilled shooters under pressure but are significantly worse over time.

46 – 14 <u>6/10</u> 14 is for skilled shooters with time. Any other situation is best served by 6/10.

47 – 15 <u>7/19</u> 15 is for skilled shooters unless they are under pressure with 2 darts in hand. Any other situation is best served by 7/19.

7 and 11 are equivalent to 15 for the skilled shooter under pressure, however, 15 is much better with time.

48 – 16 <u>16/8</u> 16 is for skilled shooters unless they are under pressure and in this and any other case the target should be the 16/8 wire. Watch out for T16 bust.

49 – 17 <u>9</u> 17 is negligibly better for players with skill. 9 is negligibly better for less skilled players. T17 brings in bust risk. Nevertheless, they are both very close.

9 and 13 offer the skilled player similar success under pressure but much worse success over time than 17. People sometimes argue against 17 because of bust risk…..it's there but this is still best option for skilled shooters.

50 – 18 <u>10</u> 18 is negligibly better for players with skill. 10 is negligibly better for less skilled players. T18 brings in bust risk. Nevertheless, they are both very close.

Skilled shooters should not be concerned about the bust risk on 18, the risk is acceptable for the benefit in the long run out probability. 10 does not guarantee an out by any incremental probability over 18 to the skilled shooter.

51 – 19 <u>11</u>   19 is negligibly better for players with skill. 11 is negligibly better for less skilled players. T19 brings in bust risk. Nevertheless, they are both very close.

For the skilled shooter, I am aware of the bust risk on 19 and I compute that the risk is acceptable for the benefit of long term out probability. 11 is not a guarantee over 19 either.   I compute that 11 is .5% better for the skilled shooter under pressure. And 19 is 4.3% better over time.   I stick with 19.

52 – 20 <u>12</u>   20 is negligibly better for players with skill. 12 is negligibly better for less skilled players. T20 brings in bust risk. Nevertheless, they are both very close.

A mathematically superior shot for the unskilled player is actually 16 instead of 12. However, the benefit is negligible and the 12 leaves a more common out target….double tops. Also the pattern is easier to remember between 50-60 outshots for the unskilled player if we keep the targets in order.   Here I took the liberties to keep the outshots simpler because the mathematically optimal outshot did not add a noticeable benefit. (Benefit is about 2% at most in situation with 3 darts and pressure….rest of the time 16 is more equivalent or actually worse.)

53 – 13   Practically optimal in all scenarios.

Sometimes 17 is argued for. There is no materially significant benefit of 17 over 13 in any situation. Really only best for unskilled players and only when they have 2 darts and pressure because the 17 has the adjacent 3 which would leave DB for the novice. But at this skill level this benefit is very small. Not worth complicating the sheet.

54 – 14   Practically optimal in all scenarios.

Sometimes it is argued to go 18 with 2 darts. I disagree. 18 only benefits the skilled shooter with 2 darts in hand and pressure. This is because a miss into 4

leaves a DB finish attempt. And this benefit is only a .6% increase in out success rate. And this comes at the cost of a 1.5% decrease in success rate over time. Not worth even considering. 18 is equivalent to 14 at best and weaker at worst.

55 – 15   Mathematically optimal in all scenarios.

56 – 16   Mathematically optimal in all scenarios.

Note: There is no benefit of going at the T16 here. Just go at the fat part of 16.

57 – 17   Practically optimal in all scenarios. With time, a B for the skilled shooter is optimal. Bull puts you on 32. However, this route comes at a significant sacrifice of closing probability on current turn. 17 is far better. I would only experiment on Bull if you have plenty of time.

58 – 18   Mathematically optimal in all scenarios.

59 – 19   Mathematically optimal in all scenarios.

60 – 20   Mathematically optimal in all scenarios.

61 – B   Practically optimal in all scenarios. T15 only marginally better with skilled shooter and time. I know I'm gonna get a lot of hate on this one for not singling out the T11 with two darts in hand. Many shooters love this T11 because of the "Second chance" on Bull if triple is missed and target single is hit. I agree with this technique for most numbers between 60-70 but not for 61, 63, or 65. Fact is, bull is the largest target that immediately puts you on an out and any miss will likely get you under 60 for an easy two dart out.

Mathematical Proof:

I'm going to show that the probability of finishing 61 starting with bull is always greater than the probability of finishing 61 starting with T11 when there are 2 darts in hand.

Definitions:

P(T) = Probability of hitting a treble

P(D) = Probability of hitting a double

P(B) = Probability of hitting a single bull (B)

P(E) = Probability of hitting a double bull (E)

P(S) = Probability of aiming at treble and missing into target single (S)

Probability (of finishing 61 starting with bull and 2 darts) = Probability of hitting a single bull X Probability of hitting next double. B D18

        So we can write

Probability (of finishing 61 starting with bull and 2 darts) =
P(B) P(D)
Or using shorthand notation
BD

Probability (of finishing 61 starting with T11 and 2 darts) = Probability of hitting T11 X Probability of hitting D14 + "Second Chance" Probability of aiming for T11 but hitting single 11 X Probability of hitting DB. Here there are two paths to closing, T11 D14 or 11 DB.

        So we can write

Probability (of finishing 61 starting with T11 and 2 darts) =
P(T) X P(D) + P(S) X P(E)
Or using shorthand notation
TD + SE

Going to prove that:
Probability (of finishing 61 starting with bull and 2 darts) >
Probability (of finishing 61 starting with T11 and 2 darts)

Similarly with shorthand notation going to prove that:
BD > TD + SE

Rearranging terms

BD – TD > SE

Grouping Terms

D (B – T) > ES

We know from the geometry of the board that the area of the single Bull (B) is
1.05 sq inches and that the area of the triple segment (T) is .4 sq inches.
Assuming a roughly uniform density probability of misses over areas roughly that
size we can relate the probability of hitting a single bull(B) vs. hitting a triple (T) to
the areas of a single bull vs. a triple. The area of the single Bull is 2.625 times the
area of the a triple. So we can say that the probability of hitting a single bull is
2.625 times the probability of hitting a treble. Notationally we can say.

B= 2.625 T

Substituting for B

D(2.625T – T) > ES

D(1.625T) > ES

1.625 X D X T > ES

Dividing both sides by E which is allowed without changing signs of the equality
because we know E to be positive

(1.625/E) X (D/E) X (T/E)   >   S

Because we know E is a positive probability between 0-1, we know that this first term is greater than or equal to 1.625.   1.625/E >=1.625

Because we know the area of D is greater than the area of E and likewise the probability of D is greater than E, we know second term D/E >1

Likewise, because the treble is larger than the double bull, we know third term T/E > 1

So product

(1.625/E) X (D/E) X (T/E)   >=   1.625

And 1>=S>=0 , because S is a probability and therefore must always be bounded by 0 and 1.

So regardless of the actual percentages for S the following identity will always hold.

(1.625/E) X (D/E) X (T/E)   > = S

So for this equality to hold, the first original equality must hold as well.

Therefore we have proven that probability of closing 61 by starting on bull is always greater than closing 61 starting with T11 with 2 darts.

Note: The bull route also has the added advantage of having its own "second chance" by missing the bull attempt into the single 11. This would only strengthen the Bull route argument.

62 – T10 *T12* T10 is practically optimal for every situation with 3 darts. Weaker shooters do negligibly better with some other options but not so much as

worth complicating the sheet. T12 with 2 darts is absolutely the best option because of the "Second Option" on Double bull.

Note: You may be saying here "Wait Andrew I thought you just got done saying you don't like the "Second Option" route?". This is not exactly what I said……I LOVE second options if they are free….I don't like second options if I have to sacrifice too much to get them. In this situation, we have to hit a triple and double to finish anyway. Why not go for the triple that leaves you the "Second Chance"? T14 D10 is just as hard as T18 D4 and just as hard as T16 D7 and just as hard as T12 D13…..only difference is with a missed T12 into 12, the shooter still has the total freebie chance of shooting at the DB to win. This is very different from the situation with 61 where you were sacrificing a much larger 1st dart target for the benefit of the second chance. This is a huge cost to pay and a cost we proved earlier is not worth the benefit. Here there is no cost to receiving the benefit.

With time and 3 darts for skilled shooters a marginally better controversial option is the 15/D15 wire. D15 puts you on 32 and a miss leaves you an easy two dart out.

63 - B   See 61……same reason.

T17, T13, T9 are all good options. B is marginally equivalent or superior in all situations. Especially situations with 2 darts and/or pressure. There is a mathematical argument to go T17, T13, or T9 with 3 darts but it is a very marginal argument. Not much extra benefit. It's about .5% difference improvement at most. I keep it simple and stick with B.

64 – T16 *T14*   T16 works in all scenarios with 3 darts. Likewise, T14 works in all 2-dart scenarios because of the "Second Chance" option on double bull. With time and 3 darts for skilled shooters a marginally better controversial option is the D16/16 wire. D16 puts you on 32 and a miss into the single leaves 48….a nice out.

65 - B   See 61……same reason.

Some argue T11 but this is inferior with 2 darts….see 61. Also this option is inferior in all situations to Bull in all situations except with a skilled shooter and 3 darts and time, but even in this situation they are basically identical probability. T15 and T19 are sometimes mentioned but these are even more so inferior.

66 – T14 *T16* 	T14 is practically optimal in all scenarios with 3 darts. Likewise, T16 works in all 2-dart scenarios because of the "Second Chance" option on double bull. With time and 3 darts for skilled shooters a marginally better controversial option is the D17/17 wire. D17 puts you on 32.

Some argue for T10 but the math shows that for unskilled shooters T10 is arguably only negligibly better than T14. For skilled shooters, T14 is materially better than T10…. especially with time. T18 is tossed out there as an option but this route sacrifices too much opportunity with time.

With time and 3 darts for skilled shooters you could go DB, 14, or 16 as well. All leading to likely 3 dart finishes.

67 – T9 *T17* 	T9 is practically optimal in all scenarios with 3 darts and especially with pressure. Any miss puts us on an easy two dart out. T17 does not offer as sweet of a contingency. A miss on either side of 17 leaves a much more difficult 2 dart out because you are not below 60. The T17 is however another fair option for the skilled shooter with plenty of time.

T17 works in all 2-dart scenarios because of the "Second Chance" option on double bull. You'll notice this is the first 60-odd that we are not shooting at bull with first dart. This is because bull does not put us on an out.

68 – T12 *T18* 	T12 is practically optimal in all scenarios with 3 darts. This option is often overlooked and it shouldn't be. T18 works in all 2-dart scenarios because of the "Second Chance" option on double bull.

With 2 darts, the controversial yet optimal option for the good shooter under pressure is the D18/18 wire. D18 puts you on 32. 18 leaves a shot at DB. This is

not on the chart because the marginal benefit of about 9% is only to the skilled shooter and only under pressure and only with 2 darts. This is not enough of a benefit in this one scenario to deal with all the drama and/or complicate the sheet.

I prefer T12 to T20 for skilled shooters because with pressure they are equivalent and with time T12 is much better.   T20 is negligibly best for all under pressure. Not so much so to warrant a spot on the chart though.

T16 is another decent option, it is a very close second to T12 for good shooters and less so for unskilled shooters. T12 is superior, this is mostly explained by the fact that 56 (assuming going 16 route) is easier to close than 52 (assuming going 20 route).   52 is encountered more often with T16 route; 56 is encountered more with T12 route. Overall, T12 (56) offers slightly better closing contingencies/probabilities over T16 (52). The graphic below shows the success probabilities of these two approaches for the skilled shooter with 3 darts over one turn. Similar, yet much lengthier and not presented, results occur for the three turn analysis.

	68 - Aiming at T16		68 - Aiming at T12
11.354%	Hit 16, w 52 hit 20, w 32 hit D16	11.354%	Hit 12, w 56 hit 16, w 40 hit D20
5.513%	Hit T16, w 20 hit D10	5.513%	Hit T12, w 32 hit D16
2.935%	Hit T16, w 20 hit 10 (D10 miss), w 10 hit D5	2.935%	Hit T12, w 32 hit 16 (D16 miss), w 16 hit D8
1.223%	Hit nextdoor 8, w 60 hit 20, w 40 hit D20	1.223%	Hit nextdoor 9, w 59 hit 19, w 40 hit D20
1.006%	Hit T16, w 20 miss off board (D10 miss), w 20 hit D10	1.006%	Hit T12, w 32 miss off board (D16), w 32 hit D16
0.886%	Hit nextdoor 7, w 61 hit B, w 36 hit D18	0.886%	Hit nextdoor 5, w 63 hit B, w 38 hit D19
0.489%	Hit nextdoor T8, w 44 hit 12, w 32 hit D16	0.489%	Hit nextdoor T5, w 53 hit 13, w 40 hit D20
0.489%	Hit nextdoor T7, w 47 hit 15, w 32 hit D16	0.489%	Hit nextdoor T9, w 51 hit 19, w 32 hit D16
0.196%	Hit 16, w 52 hit D20 (20 miss), w 12 hit D6	0.196%	Hit 12, w 56 hit D16 (16 miss), w 24 hit D12
0.028%	Hit T16, w 20 hit nextdoor D6 (D10 miss), w 8 hit D4	0.028%	Hit T12, w 32 hit nextdoor D8 (D16 miss), w 16 hit D8
0.010%	Hit nextdoor 7, w 61 hit 11 (B miss), w 50 hit DB	0.010%	Hit nextdoor 5, w 63 hit 13 (B miss), w 50 hit DB
0.021%	Hit nextdoor 8, w 60 hit D20 (20 miss), w 20 hit D10	0.196%	Hit 12, w 56 hit T16 (16 miss), w 8 hit D4
0.018%	Hit 16, w 52 hit nextdoor D1 (20 miss), w 50 hit DB	0.138%	Hit T12, w 32 hit nextdoor 8 (D16 miss), w 16 hit D8
0.008%	Hit nextdoor T7, w 47 hit T15 (15 miss), w 2 hit D1	0.033%	Hit 12, w 56 hit nextdoor T8 (16 miss), w 32 hit D16
0.008%	Hit nextdoor T8, w 44 hit T12 (12 miss), w 8 hit D4	0.033%	Hit 12, w 56 hit nextdoor D8 (16 miss), w 40 hit D20
0.008%	Hit nextdoor T8, w 44 hit D12 (12 miss), w 20 hit D10	0.028%	Hit T12, w 32 hit nextdoor D7 (D16 miss), w 18 hit D9
0.002%	Hit nextdoor 8, w 60 hit nextdoor D5 (20 miss), w 50 hit DB	0.021%	Hit nextdoor 9, w 59 hit T19 (19 miss), w 2 hit D1
0.001%	Hit nextdoor T8, w 44 hit nextdoor D5 (12 miss), w 34 hit D17	0.004%	Hit nextdoor 9, w 59 hit nextdoor T7 (19 miss), w 38 hit D19
0.001%	Hit nextdoor T8, w 44 hit nextdoor D9 (12 miss), w 26 hit D13	0.002%	Hit nextdoor 9, w 59 hit nextdoor T3 (19 miss), w 50 hit DB
24.198%		24.581%	

For unskilled shooters with three darts, the best play is actually to shoot at the single 20. This would set up the 48, an easy out that can be taken out with wire play on 16/8 to get to either D16 or D20. This was left off the chart because only in this particular case is the benefit only about 1% with time and about 4% with pressure. Not worth complicating the sheet.

69 – T11 *T19*   T11 is practically optimal in all scenarios with 3 darts. This is fairly controversial and you will almost always see T15 recommended instead of T11. T15 for skilled shooters is a very close second in all situations but this is not the optimal play. The main reason I shoot at T11 is because of the misses into adjacent singles.

When shooting at T11 and missing into 14 and then going 15 D20, this is the exact same probability as shooting at T15 and missing into the 10 and then going 19 D20.

The benefit of T11 arises when you analyze what happens when missing into the single on the other side targets just discussed. When shooting at T15 and missing into the 2, you are left with 67 which is T17 D8 with a second chance option on DB. However, when shooting at T11 and missing on the other side into the 8, you are left with 61. With two darts in hand this is B D18. 61 with two darts is better than 67 with two darts. 67 does not have a closing route that starts on B with two darts, so you have to go for the triple. If you revisit the mathematical proof for 61, you will see that I've already proven that the B route is preferable with 2 darts than going for the triple despite the triple route having the second chance option on DB.

With 69 to go, T19 works in all 2-dart scenarios because of the "Second Chance" option on double bull.

70 – T10 *T20*   T10 is practically optimal in all scenarios with 3 darts.
Likewise, T20 works in all 2-dart scenarios because of the "Second Chance" option

on double bull. With time and 3 darts for skilled shooters a marginally better controversial option is the D19/19 wire. D19 puts you on 32.

T18 is a very good alternative for the skilled shooter…..equivalent to T10. However, T18 is not as good at T10 for the less skilled shooter so I leave it off the out chart. T10 suffices all.

71 – T13 *T19*　　　　　　T13 is practically optimal in all scenarios. Only other option to consider is T11 and perhaps T11/T14 for unskilled shooters with 3 darts and pressure. Considering this is a unique situation and a rather funky recommendation with marginal benefit….I vetoed it.

With 2 darts T19 is slightly marginally better for the freebie on bull if you miss into the T7.

72 – T16　　　　　　T12, a popular option, is marginally worse for skilled players than T16 and magnitudes worse for unskilled players. With time and 3 darts for skilled shooters a marginally better option is D20/20.   D20 puts you on 32.

T16 beats T12 and T20. T20 is marginally better over time but you give up a disproportionate amount of opportunity near term.

73 – T19　　　　　　T19 is mathematically optimal for skilled shooters in all situations.   T19 is practically optimal for unskilled shooters.

74 – T14 *T16*　　　　　　T14 is practically optimal in every scenario. With pressure an interesting option to try is D17/17 or D19/19 wires. This is a marginally better solution for the current turn. A double puts you on a good out and a miss into the single leaves an easy two dart out.

T18 is marginally worse than T14 in all scenarios. I don't believe T18 should ever be used.

T16 is inferior to T14. But with 2 darts it offers you the freebie of hitting the T8 and leaving bull.

75 – T17   T17 is mathematically optimal for skilled shooters in all situations.   T19 is practically optimal for unskilled shooters. However, it is my believe that in only one situation is the T19 materially better for the unskilled player over T17 and that is with pressure and three darts. Because the T19 is only significantly better than T17 in this one scenario and by only about 12%, I decided to leave it off my sheet.

Bull math doesn't work out. Better going T17.

76 – T20   T20 is mathematically optimal in all situations.

People sometimes stress out about side risk on 20 not getting you under 70 with the first dart. A lot of these guys opt for T16 instead. Feeling this is better. However, T16 is inferior in all scenarios except the scenario of an unskilled shooter with 3 darts and pressure. In this scenario T16 leads to a close 16% more often than T20. This situational benefit is not significant enough in my mind to warrant including it on the outchart.

77 – T19   T19 is practically optimal in all situations.

T15 is often tossed around and this is equivalent to T19 with 2 darts but T15 gives up a lot with 3 darts. Most notable issue is a single 15 leaves you with 62 with 2 darts. Better to have less than 60 with 2 darts.

78 – T18   T18 is practically optimal in all situations.

I've seen T14 thrown out there and for the most part it is slightly worse than T18 in most situations and magnitudes worse with a skilled shooter and three darts under pressure.

79 – T19   T19 is practically optimal in all situations.

T13 is sometimes argued for but T13 is at best comparable to T19 and in some situations magnitudes worse. There is no benefit to going T13 over T19. And for the unskilled shooter T13 is worse is all situations.

Some say T13 with 2 darts. Wrong. Not as good as T19.

80 – T20   T20 is practically optimal in all situations.

There is no benefit in any situation of using T16 over T20. More than likely gonna hurt you by leaving you on 74 with 2 darts. Even with 2 darts….there is no benefit of T16 over T20.

81 – B *T19*   With 3 darts B is the biggest target that gets you under 60. Always best option with 3 darts. T15 is marginally better with pressure and 3 darts for the skilled shooter. However, that marginal benefit is trumped by the relative strength of the B route over time.

With 2 darts, best option is always T19.   T19 in my opinion leaves a better number T15. 24 halves 4 times in a row whereas 36 halves 3 times in a row.

82 – B *T14*   B is mathematically optimal in all situations except with 2 darts when T14 is mathematically optimal. A DB here leaves you on a great out. I've seen T20 argued for but this is dominated by my recommended strategy in all situations.

83 – B *T17*   B is practically optimal in all situations except with 2 darts when T17 is mathematically optimal.

84 – B *T16*   B is mathematically optimal in all situations except with 2 darts when T16 is mathematically optimal.

I know the hate is coming on this one. Yes T20 is as great option. I prefer T16 because it is a little better than T20 in most cases.   Shooting at T16 often leads to

shooting at T18, which is more familiar and comfortable for most compared to the T14 that is often left in the T20 route.

85 – B *T15* 		B is practically optimal in all situations except with 2 darts when T15 is practically optimal. T19 is mathematically equivalent. T15 leaves D20….preferred to D14.

86 – T18 		T18 is practically optimal in all situations.

87 – T17 		T17 is practically optimal in all situations.

88 – T16 		T16 is practically optimal in all situations.

I've struggled with this one. And I am very close to saying just go T20 but T16 does look preferable. T16 or T20. Really T20 is only really significantly better with unskilled shooters under pressure. Often people try to warn off of the T16 because a single leaves you 72. Gotta take the good with the bad here and still T16 is marginally better over time and negligibly different otherwise.

89 – T19 		T19 is practically optimal in all situations. Leaves 32.

90 – T20 *T18* 		This one always puzzled me until I did my mathematics. T20 is best with 3 darts simply because all the leaves allow further opportunities….including a shot at DB. With 2 darts, T18 is equally successful as T20. However, the probability is slightly higher after closing over time because the T18 path typically or probabilistically leads to better leaves.

I want to invest some time addressing the B route that in often suggested.

With two darts and pressure, the bull route doesn't make sense because to finish the game you have to hit a DB and a double rather than a treble and a double. Far easier to do the latter.

With two darts and time, there is a 2% improvement going B vs. T18 for skilled shooters and a 3% improvement for unskilled shooters. But this benefit is negligible. This small benefit over time is, in most all feasible situations, not worth sacrificing the huge benefit in the short term.

For the skilled shooter with three darts and pressure, the B and T20 routes are equivalently optimal. With time, however, the B route is about 4% better.

For the unskilled shooter with three darts and pressure, the T20 is about 40% better. With time, however, the B route is about 5% better.

So with 3 darts, the B route is at best a few percent better than T20 depending on the situation; for the most part it is equivalent; and at worst the B route is roughly 40% worse.

So can B be a better option on 90? Yes……but only in certain situations and negligibly enough so that it is not worth complicating the sheet.

91 – T17　　　　T17 is practically optimal in all situations. Some argue bull here but it is a suboptimal play and this route does not allow a finish in 2.

92 – T20　　　　T20 is practically optimal in all situations. Some argue bull here but it is a suboptimal play and this route does not allow a finish in 2.

93 – T19　　　　T19 is practically optimal in all situations. Some argue bull here but it is a suboptimal play and this route does not allow a finish in 2.

94 – T18　　　　T18 is practically optimal in all situations. Some argue bull here but it is a suboptimal play and this route does not allow a finish in 2.

95 – T19　　　　T19 is practically optimal in all situations. Some argue bull here but it is a suboptimal play and this route does not allow a finish in 2. Only if you are a good shooter and you are under pressure with 3 darts should you go for Bull.

96 – T20　　　　T20 is practically optimal in all situations.

97 – T19   T19 is practically optimal in all situations.

98 – T20   T20 is practically optimal in all situations.

99 – T19   T19 is practically optimal in all situations.

100 – T20   T20 is practically optimal in all situations.

101 – T20 *T17*   T20 is optimal in all situations with 3 darts. T17 is optimal with 2 as it allows a close with DB.

102 – T20   T19 is optimal for good shooters and practically optimal for weaker shooters. For simplicity and to avoid a fight. T20 is close enough.

103 – T19   T19 is practically optimal in all situations.

104 – T16 *T18*   T16 is practically optimal in all situations. T19 is a close second. T18 is a distant third with 3 darts for the skilled shooter and not even on the short list for unskilled shooters. T18 leaves a shot at DB to close.

105 – T19   T19 is practically optimal in all situations.

106 – T20   T20 is practically optimal in all situations.

107 – T19   T19 is practically optimal in all situations.

108 – T20   T20 is practically optimal in all situations.

109 – T19   T19 is practically optimal in all situations.

110 – T20   T20 is practically optimal in all situations.

111– T19   T19 is practically optimal in all situations.

8/3/2018   Last night after I finished a match, a gentleman from the other team pulled me aside and said "Andrew you should be going at T17 with 111…..not T19. T17 leaves 60 which is better than 54 because you can aim last two darts at same part of the board." I have one marginal and two very strong arguments against this approach. I'll give the marginally strong argument first and then the two strong arguments. First, I don't agree with the advantage of shooting at the same area of the board. The subtle advantage the experienced shooter gains by shooting at the same area of the board is at least matched by the danger of dart traffic blocking a future target (i.e. leaving a single 20 second dart blocking the required double 20). So I would argue that a single 14 followed by a double 20, is as easy or easier (because of no darts potentially blocking the double 20) than a single 20 and a double 20. This is my marginal defense. My second defense of the T19 approach is that the single 14 has no risk of busting. Even the miss into the triple 14, the highest adjacent miss when going for single 14, leaves an out on next dart….. T19 T14 D6.   If you go the T17 20 D20 route you bring in the risk of a bust with the second dart on T20. There is no benefit to accepting this extra risk when it can easily be avoided going the T19 route. Finally, both routes have side risk into the single 3 with the first dart, which leaves 108 and no chance of closing with the two remaining darts. The 17 has 2 on the other side, which leaves you with 109 and again no chance of closing with 2 darts. So the T17 has significant side risk on both sides. The 19 on the other hand has a 7 on the other side. With the T19 route, a miss into 7 leaves the shooter with 104. 104 has a two dart out……T18 DB. So the T19 route also has less side risk, as a miss into the 7 still leaves a two dart out.

T20 route is a close second and actually slightly better with time.

112 – T20   T20 is practically optimal in all situations.

113 – T19   T19 is practically optimal in all situations.

114 – T20   T20 is practically optimal in all situations.

115 – T19 T19 is practically optimal in all situations.

116 – T20 T20 is practically optimal in all situations.

117 – T20 T20 is practically optimal in all situations. Often people argue strongly against this in favor of T19. T20 leaves a better setup than T19, these folks usually quote the benefit of the better side risk on 19. 7 leaves a 2 dart out. T20 is best.

118 – T20 T20 is practically optimal in all situations.

119 – T19 T19 is practically optimal in all situations.

120 – T20 Really any factor of 20 should be a combination of T20, D20, 20.

121 – B T17 Practically optimal. T17 is marginally better for shooters under pressure. I get heat over the next few as well. Hitting anywhere in the bull and a triple is easier than hitting two triples. Period.

122 – B T18 Practically optimal. T18 is slightly marginally better for skilled shooters under pressure but overall B is optimal if not negligibly close to optimal in most all cases. B is clearly better with less skilled players and or time.

123 – B T19 Practically optimal. If you are a novice or a skilled shooter under pressure best to go with T19. T19 is about 15% better at closing with 3 darts for the skilled shooter. However, this comes at the sacrifice of not being as good over time as B.

124 – T20 Universally optimal solution in all scenarios. Avoid B here because it puts you on 99 which requires 3 darts to finish.

125 – B B is practically optimal in all situations and yields the best opportunity for a skilled shooter to close under pressure.

People often get scared of bull. Here's the mathematical certainly about the total area of the bull (including the double bull)compared to any triple segment…..wait for it……the total bull is 3.125 times the size of any triple. If you want to close with three darts on 125 your first dart has to be either in a triple or somewhere in the bull. I've heard countless players argue the T20 route is better because it leaves 65 which if the T15 is missed into the single 15 it leaves a "second chance" at double bull to win. This is all they care about …….. the "second chance". I've NEVER….I'm talking NEVER heard anybody in the bar ever weigh the benefit of this "second chance" in terms of likelihood they will be shooting at it and probability of being successful. Furthermore, they never consider the "Cost" they are paying to have the benefit of this "second chance" on DB with the T20 & 65 route. **I have.** You're paying too much for this "second chance"…. **always start 125 on bull**. (Math talk to follow….you can skip or read if you are stubborn and love T20 on 125)

Mathematical Proof:

I'm going to show that the probability of finishing 125 starting with bull is always greater than the probability of finishing 125 starting with T20.

Definitions:
P(T) = Probability of hitting a treble
P(D) = Probability of hitting a double
P(B) = Probability of hitting a single bull (B)
P(E) = Probability of hitting a double bull (E)
P(S) = Probability of aiming at treble and missing into target single (S)

Probability (of finishing 125 starting with bull) = Probability of hitting a single bull X Probability of hitting next triple X Probability of hitting the next double + Probability of hitting a double bull X Probability of hitting next triple X Probability of hitting the next double. With either single or double bull you are left with a triple and a double to finish. B T20 D20 or E T17 D12

So we can write

Probability (of finishing 125 starting with bull) =
P(B) X P(T) X P(D)+ P(E) X P(T) X P(D)
Or shorthand BTD + ETD

Probability (of finishing 125 starting with T20) = Probability of hitting T20 X Probability of hitting next triple X Probability of hitting the next double + "Second Chance Probability" Probability of hitting T20 X Probability of aiming for T15 but hitting single 15 X Probability of hitting DB. Here there are also two paths to closing, T20 T15 D10 or T20 15 DB.

So we can write

Probability (of finishing 125 starting with T20) =
 P(T) X P(T) X P(D) + P(T) X P(S) X P(E)
Or shorthand TTD + TSE

Going to prove that:
Probability (of finishing 125 starting with bull) >
Probability (of finishing 125 starting with T20)

Similarly with shorthand notation going to prove that:
BTD + ETD > TTD + TSE

Because we know all the probabilities are positive we can divide both sides of the equation by T to remove one T from both sides.

BD + ED > TD + SE

Rearranging terms

BD – TD > SE – ED

Grouping Terms

D (B – T) > E (S – D)

We know from the geometry of the board that the area of the single Bull (B) is 1.05 sq inches and that the area of the triple segment (T) is .4 sq inches. Assuming a roughly uniform density probability of misses over areas roughly that size we can relate the <u>probabilities of hitting</u> a single bull(B) vs. hitting a triple (T) to the <u>areas</u> of a single bull vs. a triple. The area of the B is 2.625 times the area of the T. So we can say that the probability of hitting a bull is 2.625 times the probability of hitting a treble. Notationally we can say.

B= 2.625 T

Substituting for B

D(2.625T – T) > E (S – D)

D(1.625T) > E (S - D)

1.625 X D X T > E (S – D)

Dividing both sides by E because we know E to be positive

(1.625/E) X (D/E) X (T/E)   >   S-D

Because we know E is a positive probability between 0-1

We know that this first term is greater than or equal to 1.625.   1.625/E >=1.625

Because we know the area of D is greater than the area of E
And likewise the probability of D is greater than E

We know second term D/E >1

Likewise we know third term T/E > 1

So product

(1.625/E) X (D/E) X (T/E)   >=   1.625

So

1.625 >= S-D

And we know this is always true because S and D are both probabilities and therefore must always be bounded by 0 and 1. So regardless of the actual percentages for S and D we know this identity will always hold.

Therefore we have proven that probability of closing 125 by starting on bull is always better than starting with T20.

Note: Both of these approaches can also hit the 15 (T20 approach by missing counterclockwise into T5 and the Bull approach by missing bull into the single 15 wedge) both leaving a 2 dart out. Both of these outcomes have a negligibly small and roughly equivalent outcome of occurring <.1% so I left them out to keep the math simpler.

126 – T19   T19 is practically optimal in all situations and especially with a skilled shooter under pressure. A single 19 leaves a two dart out.

127 – T20   T20 is practically optimal in all situations and especially with a skilled shooter under pressure. A single 20 leaves a two dart out.

128 - T18   T18 is practically optimal and clearly better with a skilled shooter. Single 18 leaves a two dart out.

129 - T19   T19 is easy to remember and practically optimal.

130 – T20   T20 is practically optimal in all cases and is handily the best option under case of a skilled shooter under pressure. Reason is 70 is much better with 2 darts than 73 because of the additional DB option (See 70 with 2 darts).

131 – T19   T19 is easy to remember and practically optimal.

132 – B   Same reason as 135.

133 - T19   T19 is easy to remember and practically optimal.

134 - T20   T20 is easy to remember and practically optimal

135 - B   Frankly, T19, T18, T20, B are all equivalent over time and skill and number of darts in hand….but B takes the edge when you are skilled and under pressure. You are twice as likely to close aiming at B in your first turn vs aiming at anything else. Single bull is the largest target that leaves you a two dart out. Enough said.