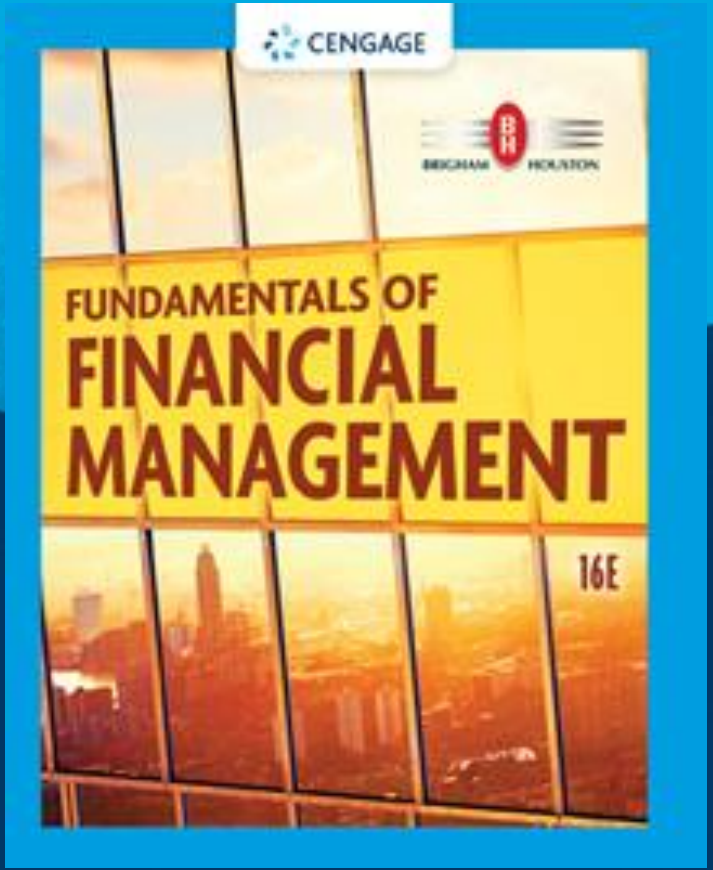
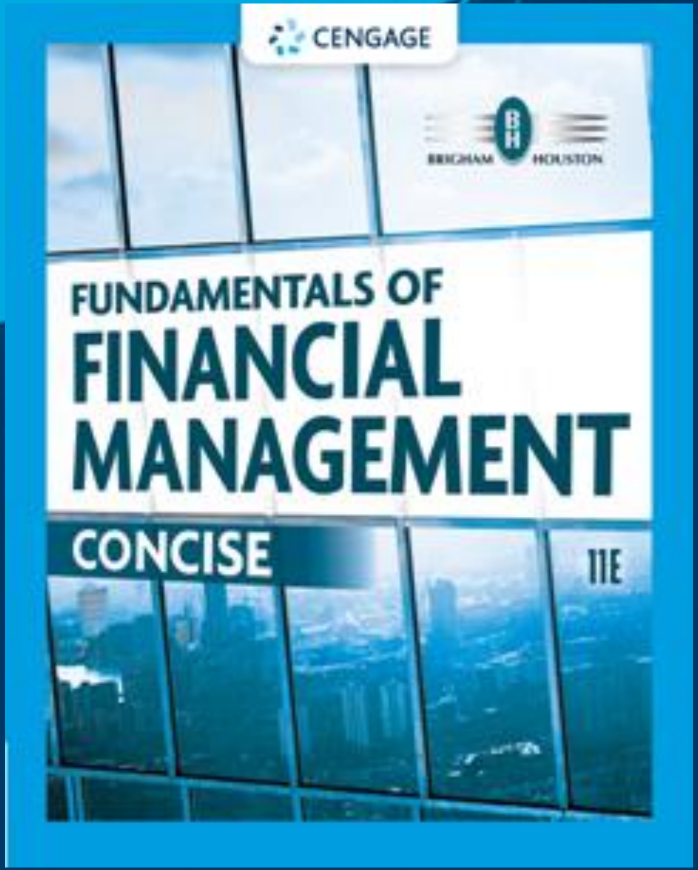




Chapter 8

Risk and Rates of Return

Overview

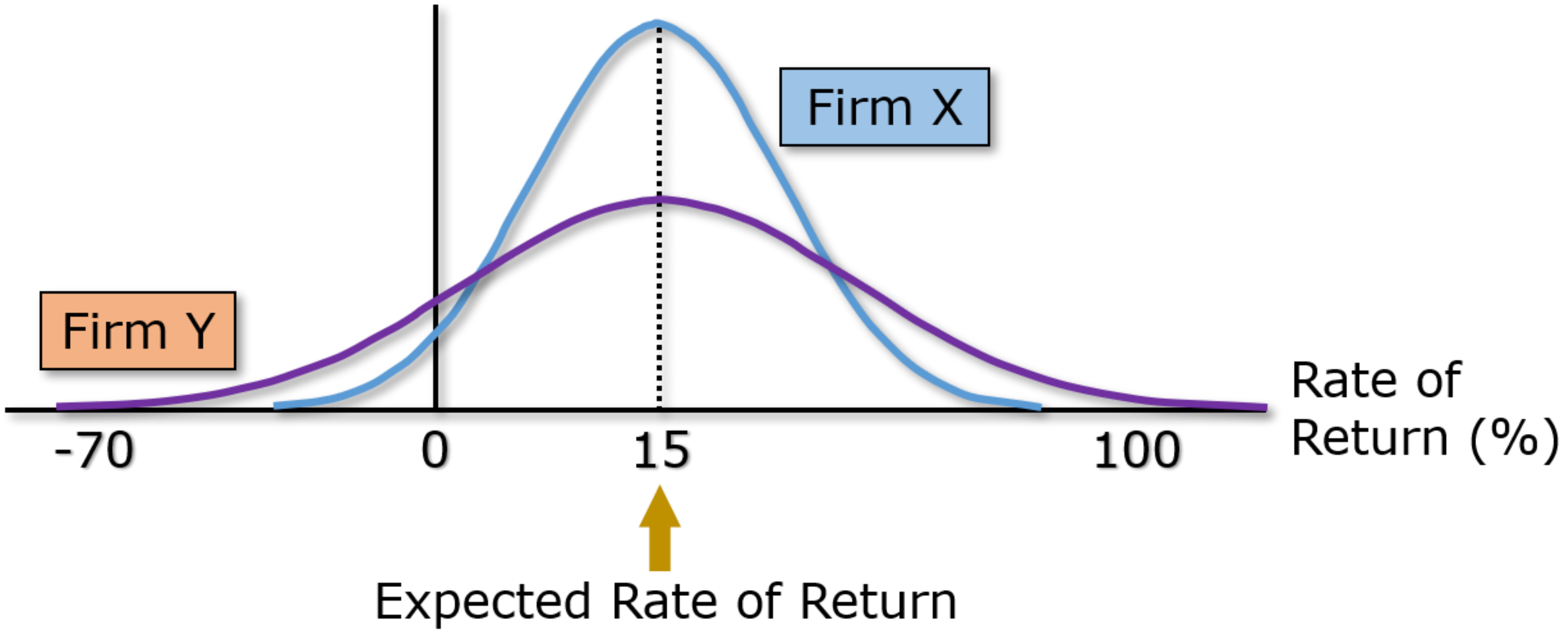
- Stand Alone Risk
- Portfolio Risk
- Risk and Return: CAPM/SML

What is investment risk?

- Two types of investment risk
  - Stand-alone risk
  - Portfolio risk
- Investment risk is related to the probability of earning a low or negative actual return.
- The greater the chance of lower than expected, or negative returns, the riskier the investment.

Probability Distributions

- A listing of all possible outcomes, and the probability of each occurrence.
- Can be shown graphically.



Selected Realized Returns, 1926-2019

	Average Return	Standard Deviation
Small-cap stocks	16.3%	31.5%
Large-cap stocks	12.1	19.8
Long-term corporate bonds	6.4	8.5
Long-term government bonds	6.0	9.8
U.S. Treasury bills	3.4	3.1
Portfolios:		
90% stocks/10% bonds	11.4%	17.8%
70% stocks/30% bonds	10.2	14.1

Source: Based on Roger G. Ibbotson, *Stocks, Bonds, Bills, and Inflation: 2020 Yearbook* (Chicago, IL: Duff & Phelps, 2020), pp. 2–6, 2–23.

Hypothetical Investment Alternatives

Economy	Probability	T-Bills	High Tech	Collections	U.S. Rubber	Market Portfolio
Recession	0.1	3.0%	−29.5%	24.5%	3.5%	−19.5%
Below average	0.2	3.0%	−9.5%	10.5%	−16.5%	−5.5%
Average	0.4	3.0%	12.5%	−1.0%	0.5%	7.5%
Above average	0.2	3.0%	27.5%	−5.0%	38.5%	22.5%
Boom	0.1	3.0%	42.5%	−20.0%	23.5%	35.5%

Why is the T-bill return independent of the economy? Do T-bills promise a completely risk-free return?

- T-bills will return the promised 3.0%, regardless of the economy.
- No, T-bills do not provide a completely risk-free return, as they are still exposed to inflation. Although, very little unexpected inflation is likely to occur over such a short period of time.
- T-bills are also risky in terms of reinvestment risk.
- T-bills are risk-free in the default sense of the word.

How do the returns of High Tech and Collections behave in relation to the market?

- High Tech: Moves with the economy, and has a positive correlation. This is typical.

- Collections: Is countercyclical with the economy, and has a negative correlation. This is unusual.

Calculating the Expected Return

$$\hat{r} = \text{Expected rate of return}$$

$$\hat{r} = \sum_{i=1}^{N} P_i r_i$$

$$\hat{r} = (0.1)(-29.5\%) + (0.2)(-9.5\%) + (0.4)(12.5\%)$$

$$+ (0.2)(27.5\%) + (0.1)(42.5\%)$$

$$= 9.9\%$$

Summary of Expected Returns

	Expected Return
High Tech	9.9%
Market	8.0%
US Rubber	7.3%
T-bills	3.0%
Collections	1.2%

- High Tech has the highest expected return, and appears to be the best investment alternative, but is it really?
- Have we failed to account for risk?

Calculating Standard Deviation

$$\sigma = \text{Standard deviation}$$

$$\sigma = \sqrt{\text{Variance}} = \sqrt{\sigma^2}$$

$$\sigma = \sqrt{\sum_{i=1}^{N} (r_i - \hat{r})^2 P_i}$$

Standard Deviation for Each Investment

$$\sigma = \sqrt{\sum_{i=1}^{N} (r_i - \hat{r})^2 P_i}$$

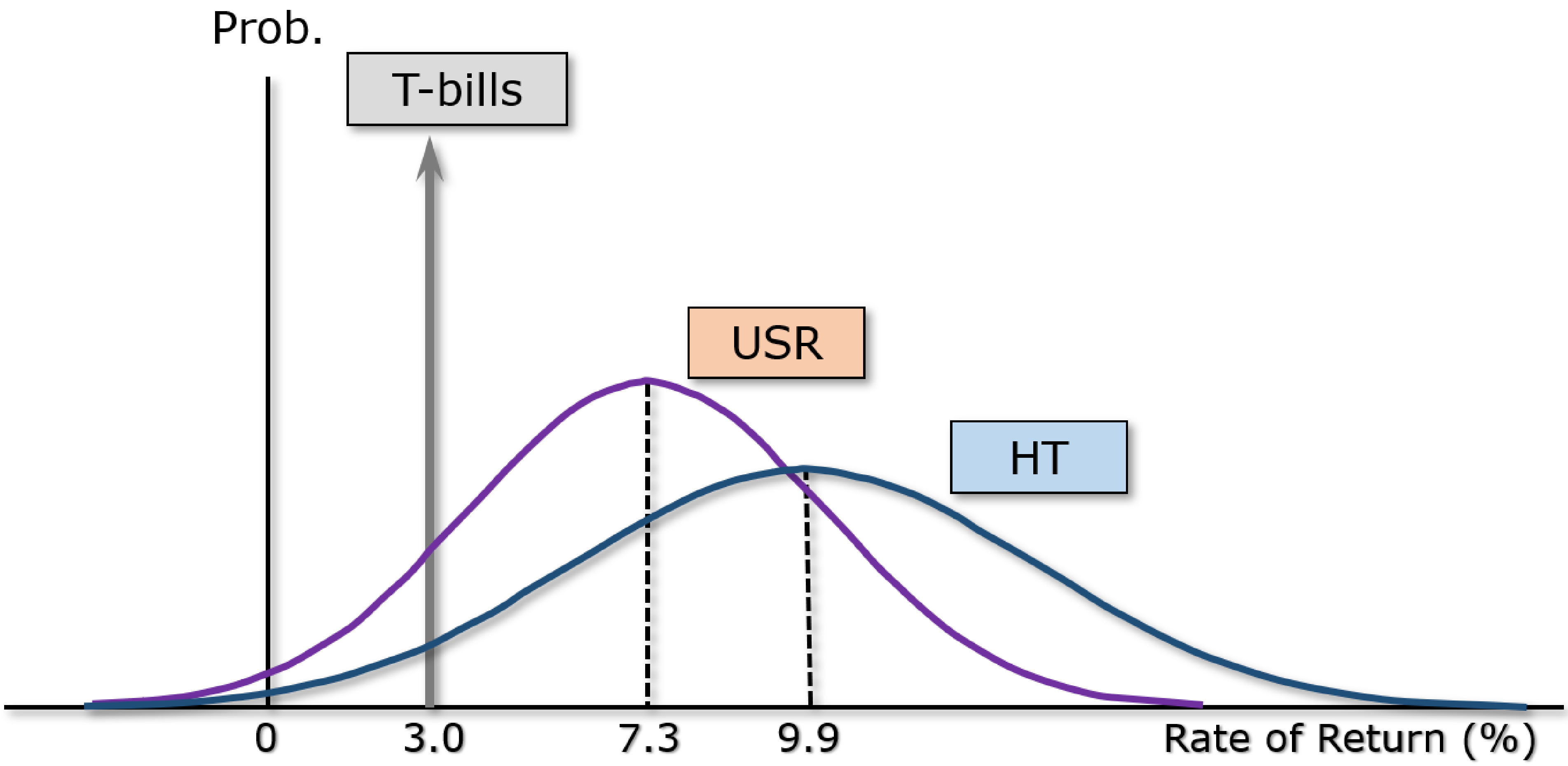
$$\sigma_{\text{T-bills}} = \begin{bmatrix} (3.0 - 3.0)^2(0.1) + (3.0 - 3.0)^2(0.2) \\ (3.0 - 3.0)^2(0.4) + (3.0 - 3.0)^2(0.2) \\ +(3.0 - 3.0)^2(0.1) \end{bmatrix}^{1/2}$$

$$\sigma_{\text{T-bills}} = 0.0\%$$

$\sigma_{HT} = 20\%$ $\sigma_{Coll} = 11.2\%$

$\sigma_{M} = 15.2\%$ $\sigma_{USR} = 18.8\%$

Comparing Standard Deviations



Comments on Standard Deviation as a Measure of Risk

- Standard deviation (σ_i) measures total, or stand-alone, risk.
- The larger σ_i is, the lower the probability that actual returns will be close to expected returns.
- Larger σ_i is associated with a wider probability distribution of returns.

Comparing Risk and Return

Security	Expected Return, $\hat{r}$	Risk, σ
T-bills	3.0%	0.0%
High Tech	9.9	20.0
Collections*	1.2	11.2
US Rubber*	7.3	18.8
Market	8.0	15.2

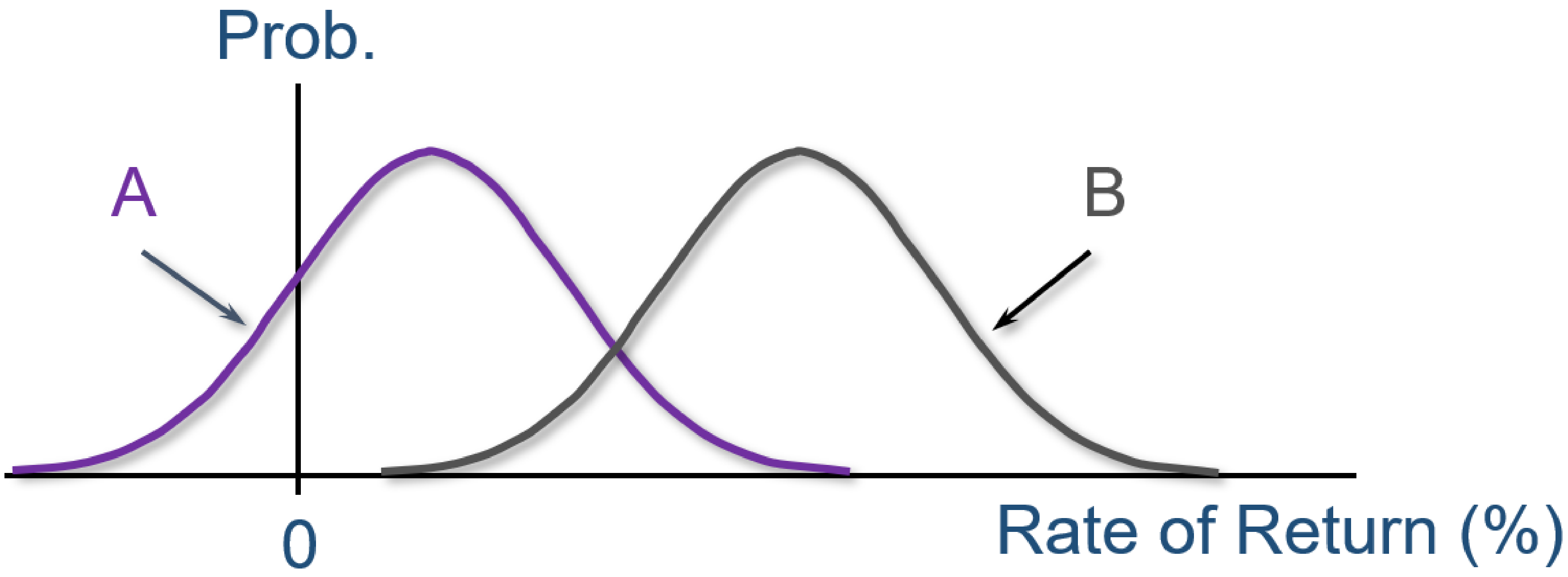
*Seems out of place.

Coefficient of Variation (CV)

- A standardized measure of dispersion about the expected value, that shows the risk per unit of return.

$$\text{CV} = \frac{\text{Standard deviation}}{\text{Expected return}} = \frac{\sigma}{\hat{r}}$$

Illustrating the CV as a Measure of Relative Risk



$\sigma_A = \sigma_B$, but A is riskier because of a larger probability of losses. In other words, the same amount of risk (as measured by σ) for smaller returns.

Risk Rankings by Coefficient of Variation

	CV
T-bills	0.0
Market	1.9
High Tech	2.0
US Rubber	2.6
Collections	9.8

- Collections has the highest degree of risk per unit of return.
- High Tech, despite having the highest standard deviation of returns, has a relatively average CV.

Sharpe Ratio

- The Sharpe ratio is an alternative measure of stand-alone risk. It looks at excess return relative to risk.
  - High Tech's Sharpe ratio = (9.9% - 3%)/20.0% = 0.345
  - U.S. Rubber's Sharpe ratio = (7.3% - 3%)/18.8% = 0.229
  - Market Portfolio's Sharpe ratio = (8% - 3%)/15.2% = 0.329
  - Collections' Sharpe ratio = (1.2% - 3%)/11.2% = -0.161
  - T-Bills' Sharpe ratio = 0.
- Excess return is asset's return minus the risk-free rate.
- Risk is measured as the standard deviation of the asset's return.
- A risk-free asset will have a Sharpe ratio = 0.

Investor Attitude Towards Risk

Risk *premium*: the difference between the return on a risky asset and a riskless asset, which serves as compensation for investors to hold riskier securities.

Risk *aversion*: assumes investors dislike risk and require higher rates of return to encourage them to hold riskier securities.

Portfolio Construction: Risk and Return

- Assume a two-stock portfolio is created with $50,000 invested in both High Tech and Collections.
- A portfolio’s expected return is a weighted average of the returns of the portfolio’s component assets.
- Standard deviation is a little more tricky and requires that a new probability distribution for the portfolio returns be constructed.

Calculating Portfolio Expected Return

$\hat{r}_p$ is a weighted average:

$$\hat{r}_p = \sum_{i=1}^{N} w_i \hat{r}_i$$

$$\hat{r}_p = 0.5(9.9\%) + 0.5(1.2\%) = 5.5\%$$

An Alternative Method for Determining Portfolio Expected Return

Economy	Probability	High Tech	Collections	Portfolio
Recession	0.1	−29.5%	24.5%	−2.5%
Below average	0.2	−9.5%	10.5%	0.5%
Average	0.4	12.5%	−1.0%	5.8%
Above average	0.2	27.5%	−5.0%	11.3%
Boom	0.1	42.5%	−20.0%	11.3%

$$\hat{r}_p = 0.10\ (-2.5\%) + 0.20\ (0.5\%) + 0.40\ (5.8\%) + 0.20\ (11.3\%) + 0.10\ (11.3\%) = 5.5\%$$

Calculating Portfolio Standard Deviation, CV, and Sharpe Ratio

$$\sigma_p = \begin{bmatrix} 0.10\,(-2.5 - 5.5)^2 \\ +\,0.20\,(0.5 - 5.5)^2 \\ +\,0.40\,(5.8 - 5.5)^2 \\ +\,0.20\,(11.3 - 5.5)^2 \\ +\,0.10\,(11.3 - 5.5)^2 \end{bmatrix}^{1/2} = 4.6\%$$

$$CV_p = \frac{4.6\%}{5.5\%} = 0.84$$

Sharpe ratio = (5.5% − 3%)/4.6%
= 0.543

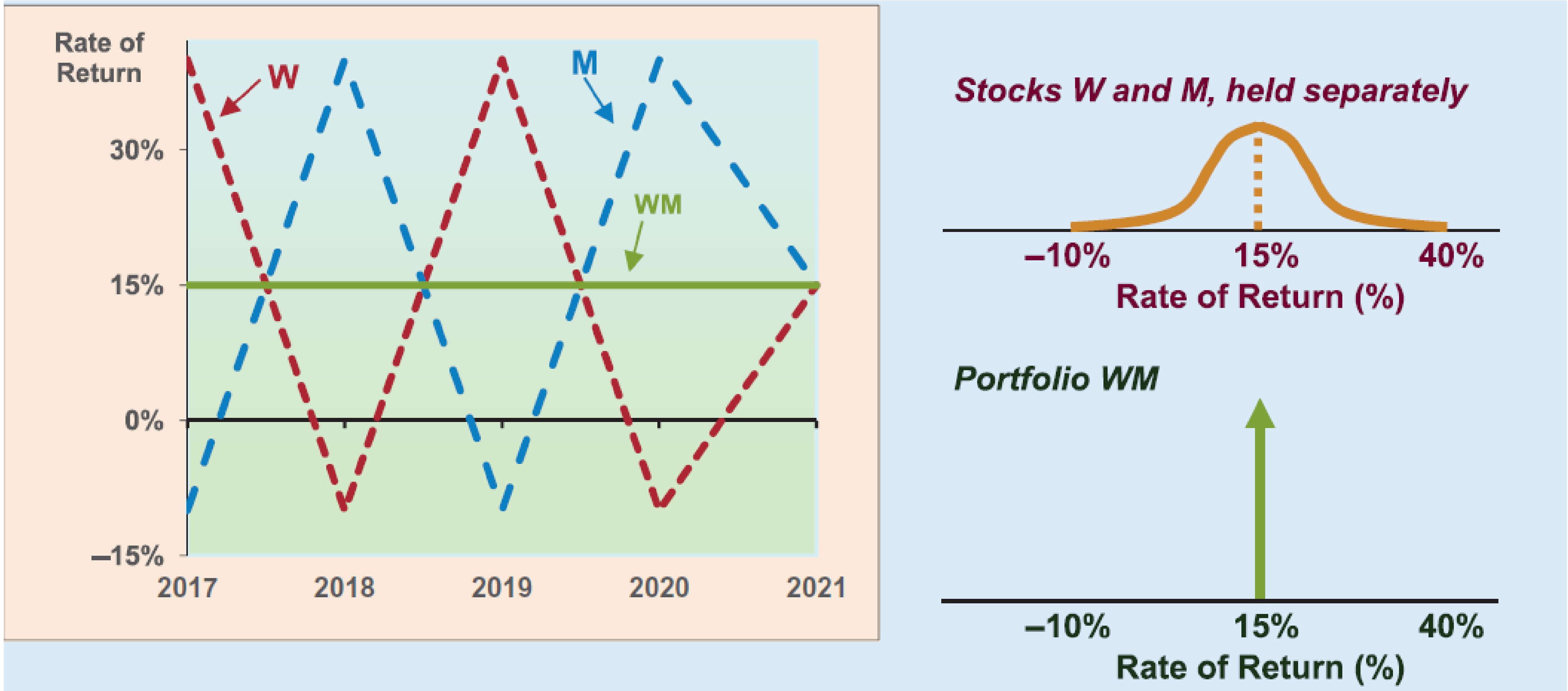
Comments on Portfolio Risk Measures

- σ_p = 4.6% is much lower than the σ_i of either stock (σ_{HT} = 20.0%; σ_{Coll} = 11.2%).
- σ_p = 4.6% is lower than the weighted average of High Tech and Collections' σ (15.6%).
- Therefore, the portfolio provides the average return of component stocks, but lower than the average risk.
- Why? Negative correlation between stocks.

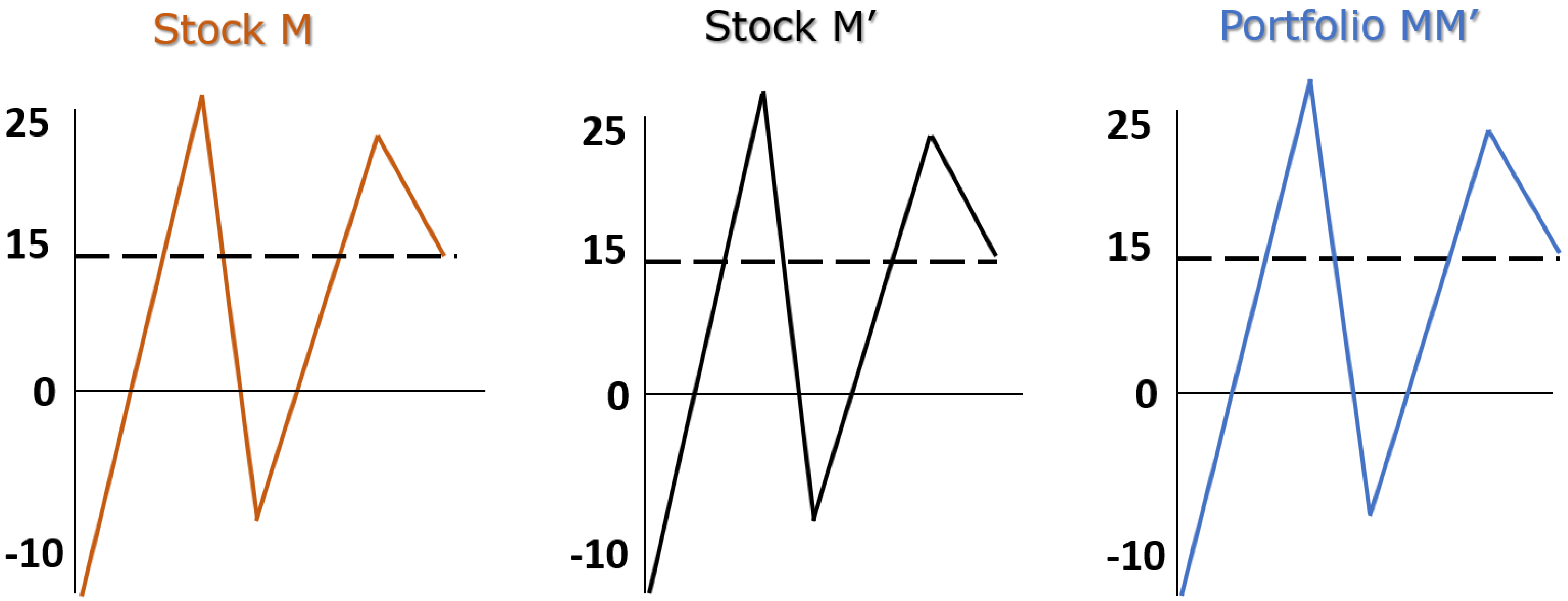
General Comments About Risk

- σ ~35% for an average stock.
- Most stocks are positively (though not perfectly) correlated with the market (i.e., ρ between 0 and 1).
- Combining stocks in a portfolio generally lowers risk.

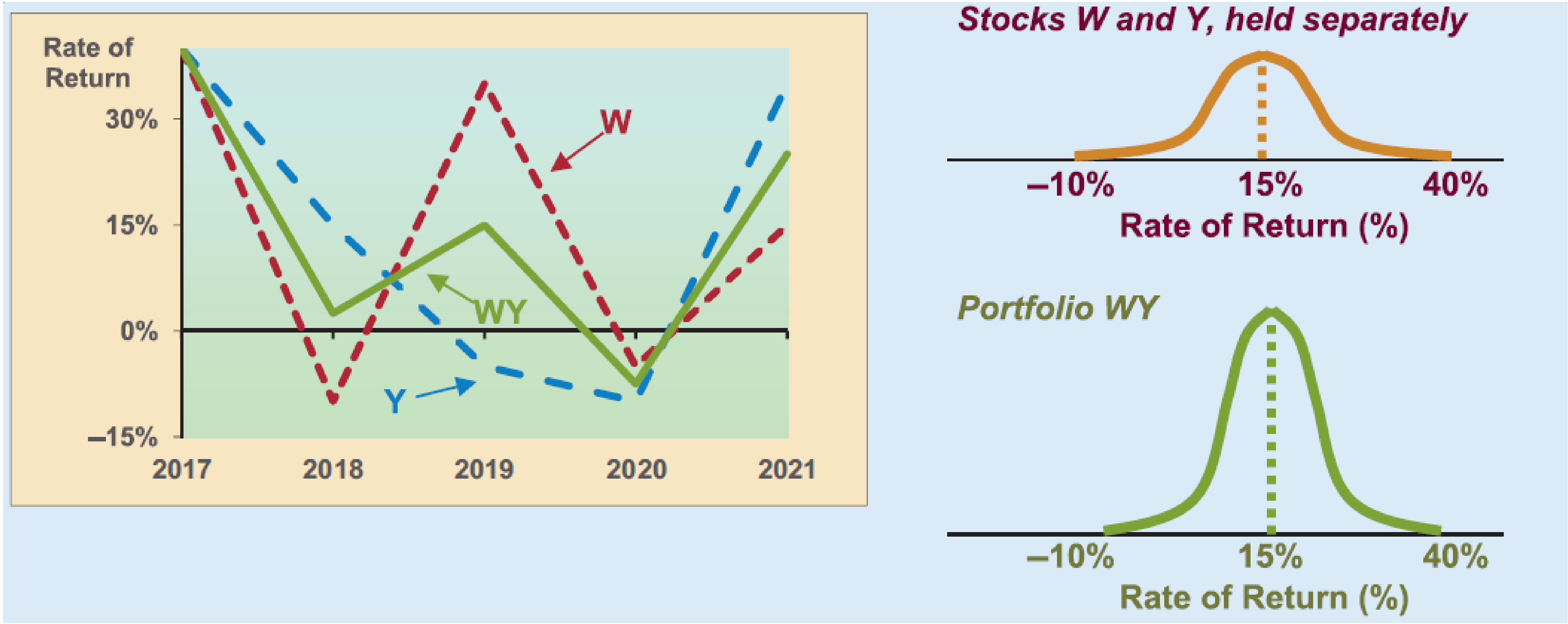
Returns Distribution for Two Perfectly Negatively Correlated Stocks (ρ = -1.0)



Returns Distribution for Two Perfectly Positively Correlated Stocks (ρ = 1.0)



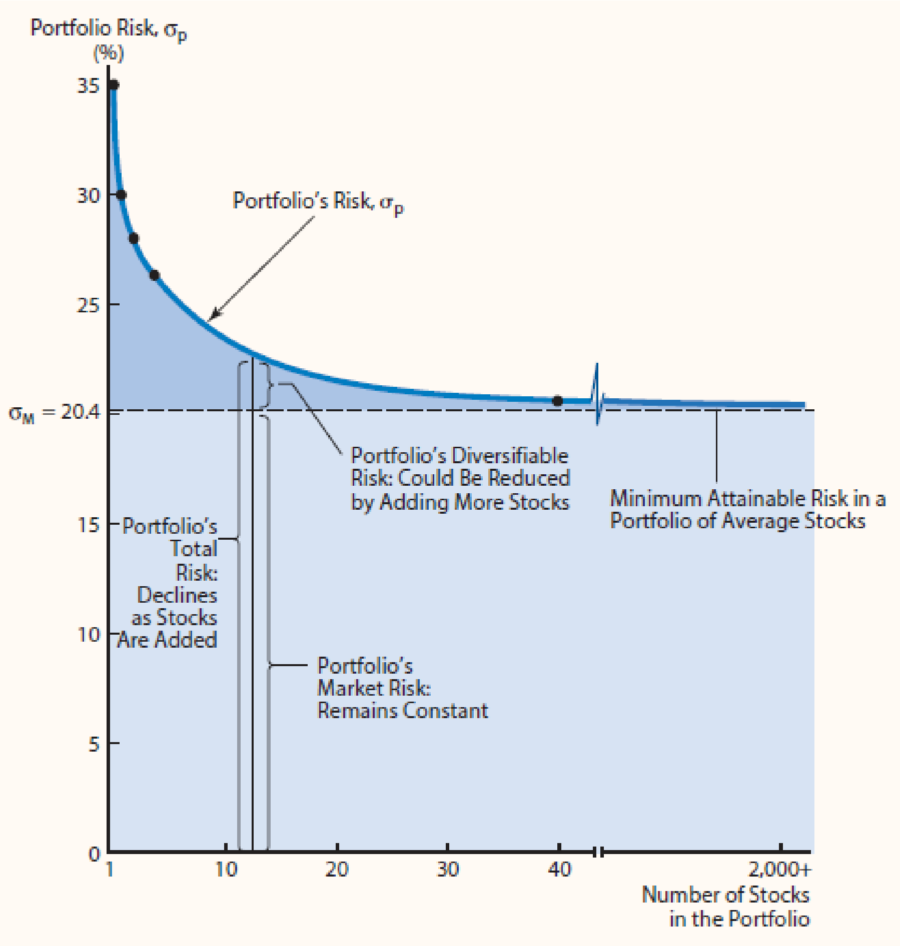
Partial Correlation, $\rho = +0.35$



Creating a Portfolio: Beginning with One Stock and Adding Randomly Selected Stocks to Portfolio

- σ_p decreases as stocks are added, because they would not be perfectly correlated with the existing portfolio.
- Expected return of the portfolio would remain relatively constant.
- Eventually the diversification benefits of adding more stocks dissipates (after about 40 stocks), and for large stock portfolios, σ_p tends to converge to $\approx$ 20%.

Illustrating Diversification Effects of a Stock Portfolio



Breaking Down Sources of Risk

Stand-alone risk = Market risk + Diversifiable risk

- Market risk: portion of a security's stand-alone risk that cannot be eliminated through diversification. Measured by beta.
- Diversifiable risk: portion of a security's stand-alone risk that can be eliminated through proper diversification.

Failure to Diversify

If an investor chooses to hold a one-stock portfolio (doesn’t diversify), would the investor be compensated for the extra risk they bear?

- NO!
- Stand-alone risk is not important to a well-diversified investor.
- Rational, risk-averse investors are concerned with σ_p, which is based upon market risk.
- There can be only one price (the market return) for a given security.
- No compensation should be earned for holding unnecessary, diversifiable risk.

Capital Asset Pricing Model (CAPM)

- Model linking risk and required returns. CAPM suggests that there is a Security Market Line (SML) that states that a stock's required return equals the risk-free return plus a risk premium that reflects the stock's risk after diversification.

$$r_i = r_{RF} + (r_M - r_{RF})b_i$$

- Primary conclusion: The relevant riskiness of a stock is its contribution to the riskiness of a well-diversified portfolio.

Beta

- Measures a stock's market risk, and shows a stock's volatility relative to the market.
- Indicates how risky a stock is if the stock is held in a well-diversified portfolio.

Comments on Beta

- If beta = 1.0, the security is just as risky as the average stock.
- If beta > 1.0, the security is riskier than average.
- If beta < 1.0, the security is less risky than average.
- Most stocks have betas in the range of 0.5 to 1.5.

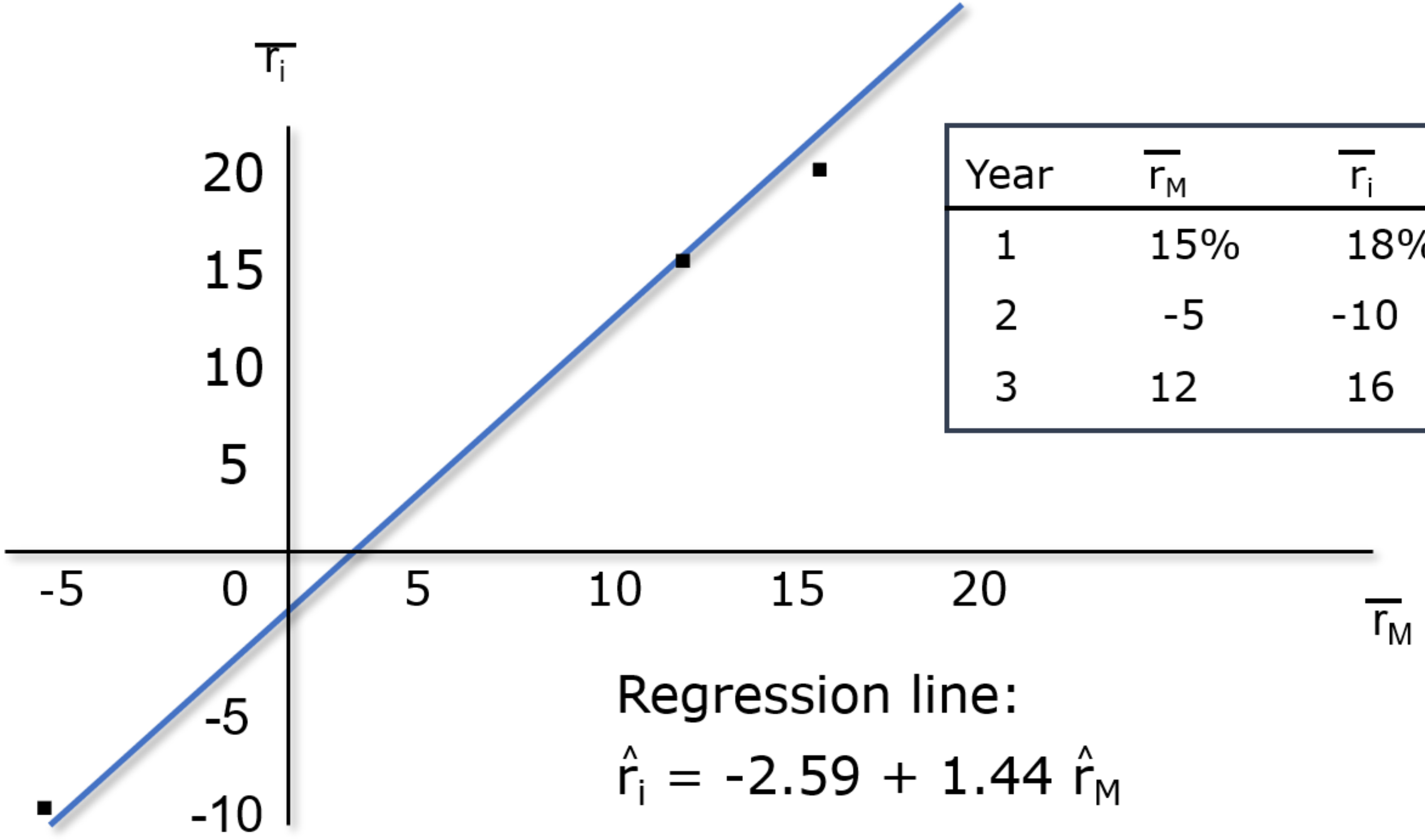
Can the beta of a security be negative?

- Yes, if the correlation between Stock i and the market is negative (i.e., $\rho_{i,m} < 0$).
- If the correlation is negative, the regression line would slope downward, and the beta would be negative.
- However, a negative beta is highly unlikely.

Calculating Betas

- Well-diversified investors are primarily concerned with how a stock is expected to move relative to the market in the future.
- Without a crystal ball to predict the future, analysts are forced to rely on historical data. A typical approach to estimate beta is to run a regression of the security's past returns against the past returns of the market.
- The slope of the regression line is defined as the beta coefficient for the security.

Illustrating the Calculation of Beta

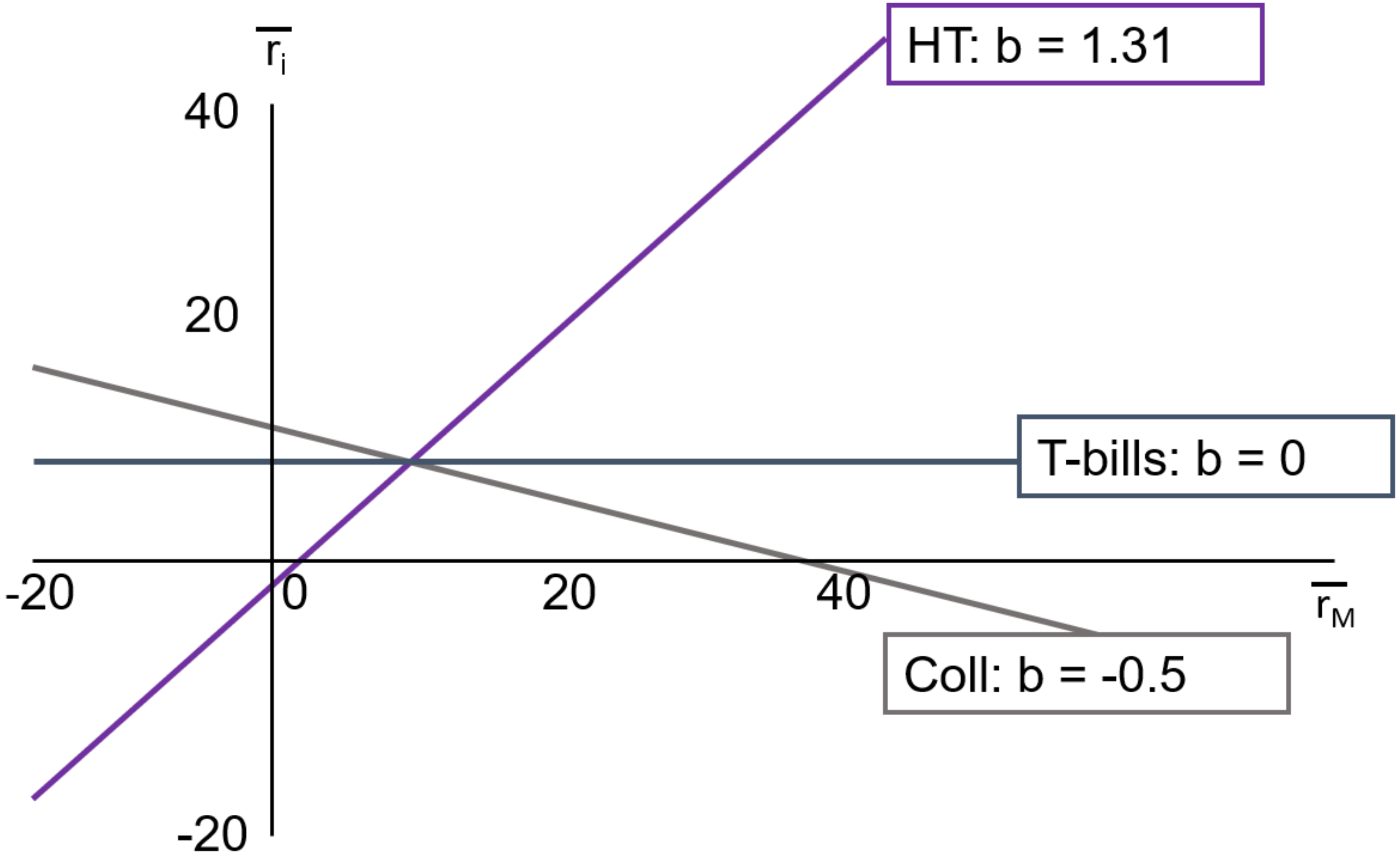


Year	$\bar{r}_M$	$\bar{r}_i$
1	15%	18%
2	-5	-10
3	12	16

Regression line:

$$\hat{r}_i = -2.59 + 1.44\ \hat{r}_M$$

Beta Coefficients for High Tech, Collections, and T-Bills



Comparing Expected Returns and Beta Coefficients

Security	Expected Return	Beta
High Tech	9.9%	1.31
Market	8.0	1.00
US Rubber	7.3	0.88
T-Bills	3.0	0.00
Collections	1.2	−0.50

Riskier securities have higher returns, so the rank order is OK.

The Security Market Line (SML): Calculating Required Rates of Return

$$\text{SML}: r_i = r_{RF} + (r_M - r_{RF})b_i$$

$$r_i = r_{RF} + (RP_M)b_i$$

- Assume the yield curve is flat and that r_{RF} = 3.0% and

$$RP_M = r_M - r_{RF} = 8.0\% - 3.0\% = 5.0\%.$$

CENGAGE

What is the market risk premium?

- Additional return over the risk-free rate needed to compensate investors for assuming an average amount of risk.
- Its size depends on the perceived risk of the stock market and investors' degree of risk aversion.
- Varies from year to year, but most estimates suggest that it ranges between 4% and 8% per year.

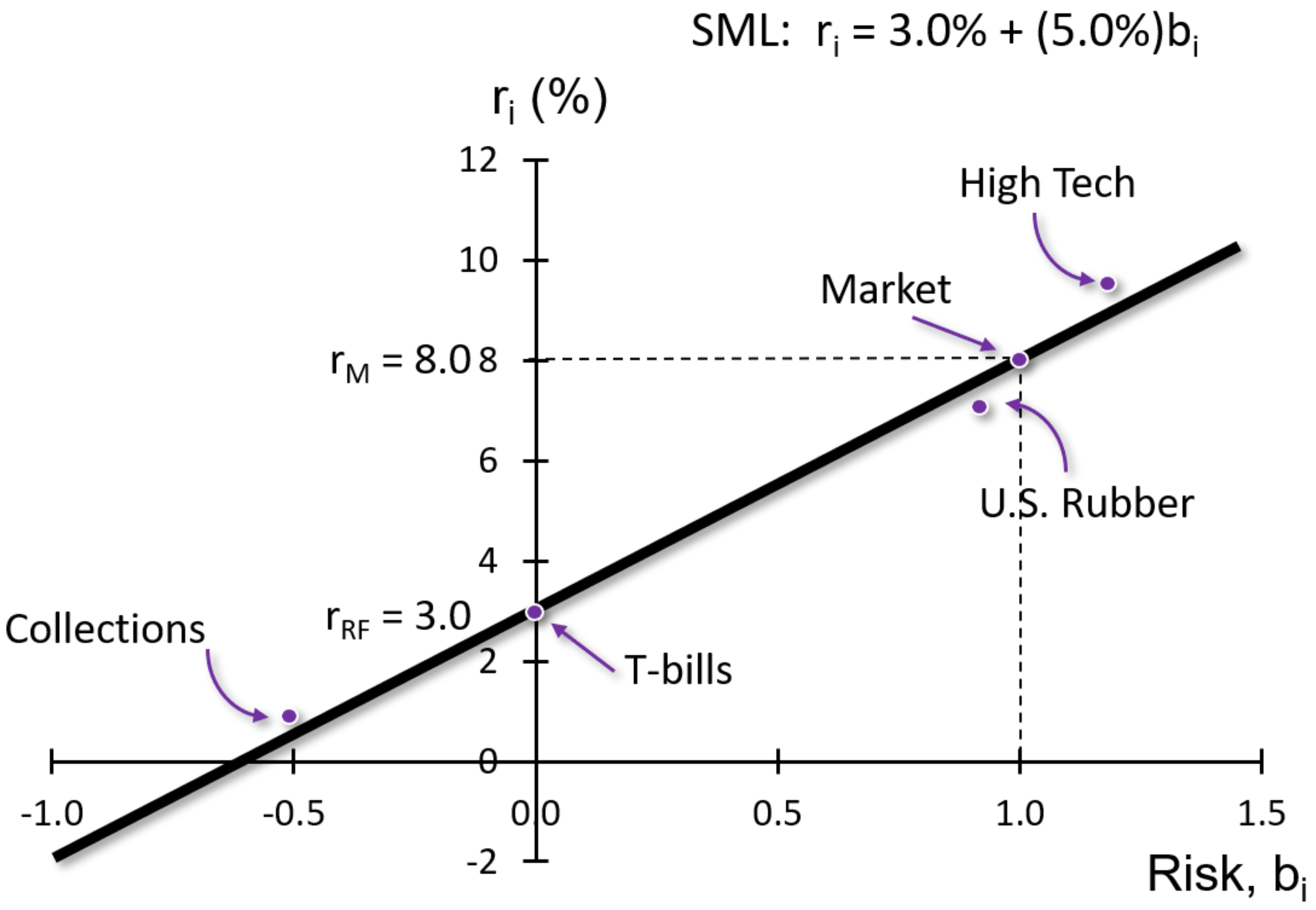
Calculating Required Rates of Return

r_{HT}	=	3.0% + (5.0%)(1.31)		
r_M	=	3.0% + 6.55%	=	9.55%
r_{USR}	=	3.0% + (5.0%)(1.00)	=	8.00%
$r_{T\text{-bill}}$	=	3.0% + (5.0%)(0.88)	=	7.40%
	=	3.0% + (5.0)(0.00)	=	3.00%
r_{Coll}	=	3.0% + (5.0%)(−0.50)	=	0.50%

Expected vs. Required Returns

	$\hat{r}$	r	
High Tech	9.9%	9.55%	Undervalued
Market	8.0	8.00	Fairly valued
US Rubber	7.3	7.40	Overvalued
T-bills	3.0	3.00	Fairly valued
Collections	1.2	0.50	Undervalued

Illustrating the Security Market Line



An Example: Equally-Weighted Two-Stock Portfolio

- Create a portfolio with 50% invested in High Tech and 50% invested in Collections.
- The beta of a portfolio is the weighted average of each of the stock's betas.

$$b_P = w_{HT}b_{HT} + w_{Coll}b_{Coll}$$
$$b_P = 0.5(1.31) + 0.5(-0.50)$$
$$b_P = 0.405$$

Calculating Portfolio Required Returns

- The required return of a portfolio is the weighted average of each of the stock's required returns.

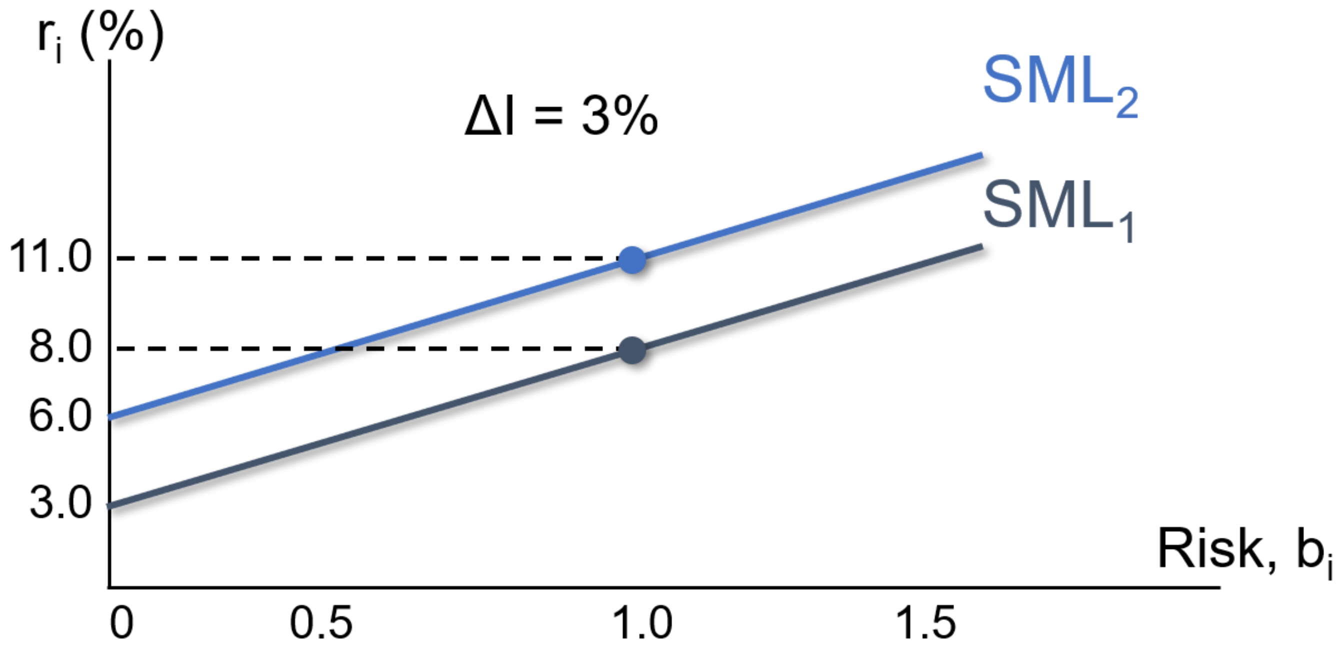
$$r_P = w_{HT}r_{HT} + w_{Coll}r_{Coll}$$
$$r_P = 0.5(9.55\%) + 0.50(0.50\%)$$
$$r_P = 5.0\%$$

- Or, using the portfolio's beta, CAPM can be used to solve for the portfolio's required return.

$$r_P = r_{RF} + (RP_M)b_P$$
$$r_P = 3.0\% + (5.0\%)(0.405)$$
$$r_P = 5.0\%$$

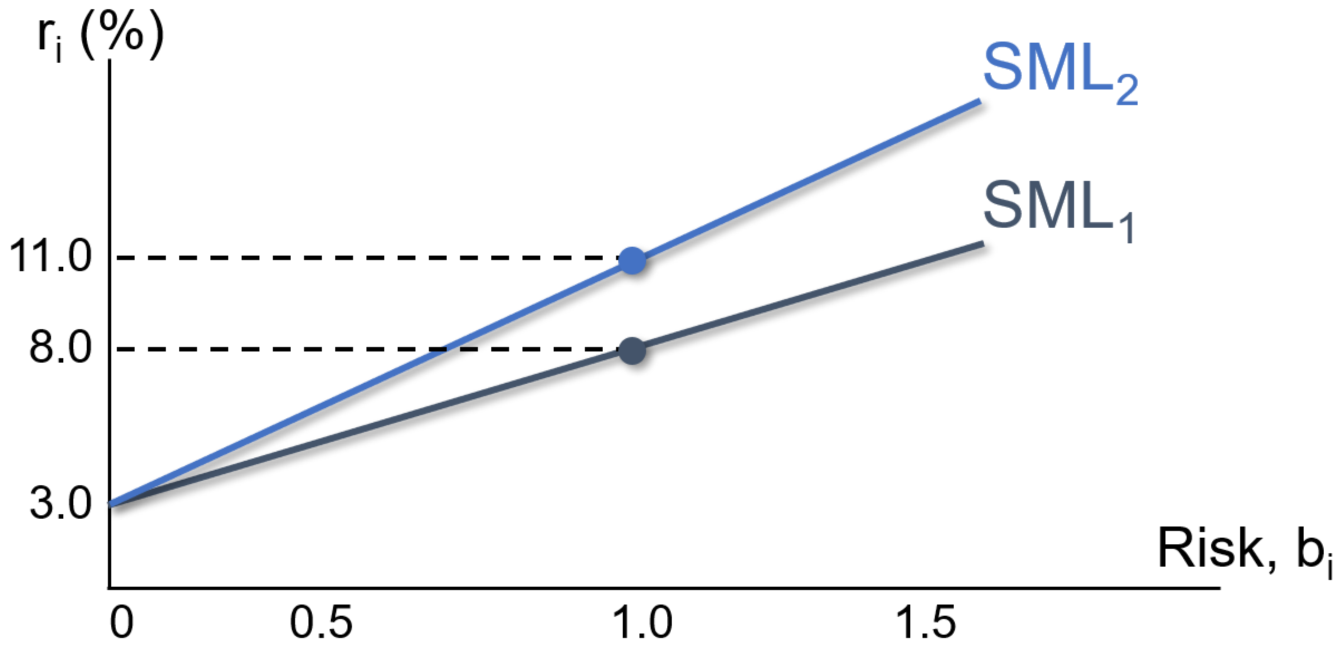
Factors That Change the SML (1 of 2)

- If investors raise inflation expectations by 3%, what would happen to the SML?



Factors That Change the SML (2 of 2)

- If investors' risk aversion increased, causing the market risk premium to increase by 3%, what would happen to the SML?



Verifying the CAPM Empirically

The CAPM has not been verified completely.

Statistical tests have problems that make verification almost impossible.

Some argue that there are additional risk factors, other than the market risk premium.