Readers' Reflections

We are glad to note that with an issue such as our last (attribution focused) we have resulting commentaries. The many facets of attribution lend this topic to on-going dialogue and challenge. Here we have two thought-provoking items, one that asks us to stop, think and regroup and another that draws us further down a known path. They are much in contrast with each other, but still very much on topic.

While We Expound on Theory, Have We Forgotten Practice?

Linking attribution effects is clearly the "hot topic" in performance right now, with a half-dozen papers on linking algorithms, followed by a second series of rebuttals, challenges and debates. Unfortunately, all this activity is generating more heat than light. It is not necessarily bad that a consensus on linking has failed to emerge, nor is it terrible that this discussion "has taken on almost a religious tone" (to quote Dave Spaulding's humorous piece in the Fall 2002 issue of *The Journal of Performance Measurement*). However, it IS a great concern that there is so little healthy skepticism among the performance analysts who read these articles, and who accept the authors' statements with "blind faith." Perhaps the performance community should adopt the same standard of scrutiny that is used in the academic and scientific communities. There, an author describes a proposed theory or methodology, and then presents the actual data used in the analysis. This allows the readers to try to replicate the author's results, followed by additional "out-of-sample" testing to affirm that the method delivers the suggested results in the various real-life scenarios that are likely to be encountered. This has not been the case with the proposed linking algorithms, which have been accepted with virtually no testing. This blanket acceptance ignores the legitimate challenge that none of these methods provides a material difference or improvement over naïve linking with simple proportional smoothing ("Is Linking Attribution Effects as Hard as it Looks" by David Spaulding, in the Spring 2002 issue of *The Journal of Performance Measurement*).

All of the algorithm authors use hypothetical, wildly exaggerated scenarios to defend their contention that their particular linking method produces unique results. These scenarios involve:

1. return differentials that would get any manager fired,

2. sector swings and turnover that would get any manager fired (after bankrupting the firm from excessive transactions costs) and

3. benchmark sector volatility that simply does not exist, since benchmarks are passive by definition.

One finds that IN REAL LIFE such scenarios are rarely, if ever encountered. As such, it is at best impractical to define complex and expensive methodologies when there is no real difference in the result.

For example, Jose Menchero's article uses the following return scenario ("An Optimized Approach to Linking Attribution Effects Over Time," *JPM* Fall 2000):

Period	Portfolio	Benchmark
1	10	5
2	25	15
3	10	20
4	–10	10
5	5	–8
6	15	–5

Assuming these are monthly returns, where has such volatility or level of return ever been experienced? Certainly not on this planet! With a correlation of only 11%, one has to wonder what the manager is doing to generate such wildly different returns, or whether the right benchmark is being used. Clearly, this client has much bigger problems than whether the attribution effects are being linked effectively!

Andrew Frongello employs both wild sector swings and the world's first "dynamic benchmark" in his illustration ("Linking Single Period Attribution Results," *JPM* Spring 2002). In his scenario (which others have quoted in subsequent articles) the portfolio's stock/bond allocation moves from 95/5 to 15/85 while the benchmark allocation moves from 10/90 to 50/50.

Perhaps the least plausible hypothetical scenario is the one offered by Menchero to illustrate the inadequacies of simple proportional smoothing ("Is Linking Attribution Effects as Hard as it Looks" by David Spaulding). At first glance, it appears reasonable:

	Port. ROR	Bench. ROR	Alpha	Alloc.	Select.
Month 1	10.0%	5.1%	4.9%	–2.0%	6.9%
Month 2	10.0%	15.0%	–5.0%	–2.0%	–3.0%

However, is this scenario typical, or is it even possible? Try to "reverse engineer" the weightings and returns that would produce these results and you will find that this scenario is not simply unlikely, it is *impossible* to replicate using a standard benchmark, realistic returns and a realistic degree of manager discretion. To create this result with a stock/bond portfolio you would need to resort to dramatically shifting the benchmark weightings between periods (52/48 to 100/0). Using a 10-sector equity portfolio, you would need to employ extreme concentration risk, avoiding half of the sectors in the benchmark. With the remaining sectors, you would need to employ huge over-weights (15x the benchmark) and under-weights (–86% less than the benchmark).

The real question is: "What results do you get when you try linking REAL LIFE data?" To answer this question, I asked one of the algorithm authors to use attribution results from his own firm's data, and to test whether the various approaches (Carino, Menchero, Frongello and naïve linking) produce materially different results. Not surprisingly, using rolling 12-month linked results over several years, the maximum difference between attribution effects averaged about 3/10ths of a basis point (one extreme example yielded a 1 bps difference). More importantly, the proportionality of the effects (the percent of alpha was attributable to allocation vs. selection) was virtually indistinguishable in all cases.

The data and analytics for these analyses are available to anyone requesting them.

Perhaps all this discussion about linking methodologies is "much ado about nothing." Perhaps our time would be better spent trying to get the alpha correct in the first place by using the appropriate risk-adjusted, style-adjusted benchmarks. It seems a bit silly to concentrate on "slicing and dicing" a number that we know to be wrong. Or, have we forgotten the big bonuses paid to growth-oriented managers during the 1995–1999 period, when even below-average managers "beat" the S&P 500? Clearly, using the right benchmark is the most important decision in attribution analysis. Only then should we worry about less-important questions such as arithmetic vs. geometric attribution, how frequently to calculate attribution, and which linking algorithm is the "right" one.

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The Recursive Family Dilemma

In the Fall 2002 issue, David Cariño wrote a very informative piece entitled "Refinements in Multi-Period Attribution." In this article, he noted a dilemma of the recursive family of methods, which include the Frongello linking algorithm. He noted that portions of outperformance arise when current attributes compound over other unique attributes earned in prior periods. For example, a portion of total outperformance occurs when Selection return in Period 2 compounds over Allocation return earned in Period 1. The dilemma – is this "cross product" portion of outperformance Allocation or Selection (or perhaps a portion of each).

I agree with Mr. Cariño that the "correct" assignment depends on which assignment the reader perceives as correct. Arguments can be made for the intuitive appeal

Table 1

To Assign the Cross Product of Selection Earned Over Allocation to:	Use the Following Formula:
Selection	Frongello Linking Algorithm $F_{tb} = G_{tb}\prod_{j=1}^{t-1}(1+R_j) + \overline{R}_t\sum_{j=1}^{t-1}F_{jb}$
Allocation	Reversed Frongello Linking Algorithm $F_{tb} = G_{tb}\prod_{j=1}^{t-1}(1+\overline{R}_j) + R_t\sum_{j=1}^{t-1}F_{jb}$
Half Allocation Half Selection	Modified Frongello Linking Algorithm $F_{tb} = G_{tb}.5[\prod_{j=1}^{t-1}(1+R_j) + \prod_{j=1}^{t-1}(1+\overline{R}_j)] + .5(R_t + \overline{R}_t)\sum_{j=1}^{t-1}F_{jb}$

of either method and this further complicates the issue. Furthermore, I would like to add for consideration that we could think of these cross product terms as a combination of each effect. Table 1[1] identifies the appropriate formula to use given the user's desired treatment of the cross product.[2]

FRONGELLO LINKING ALGORITHM

1. Current attributes are earned from the total investment base of the portfolio, regardless of how the base achieved its value. Therefore, the investment base is defined by total portfolio return.

2. All earned attributes compound into the future at the benchmark rate of return.

- *Notice*: Due to the first assumption, even if the investment base grew due to Allocation in prior periods, Selection return earned over this base is labeled Selection. Thus, the cross product is assigned to the attribute that occurred last, Selection.

REVERSED FRONGELLO LINKING ALGORITHM [3]

1. Attributes are earned only on the portion of the portfolio's investment base that has grown due to the benchmark rate of return. Therefore, the investment base is defined by the benchmark total return.

2. All earned attributes compound into the future at the total portfolio rate of return.

- *Notice*: Due to the second assumption, even if the total future return is due to Selection, the compounding of Allocation earned in prior periods into the future is labeled Allocation. Thus, the cross product is assigned to the attribute that occurred first, Allocation.

MODIFIED FRONGELLO LINKING ALGORITHM

1. Attributes are earned on a base that is defined by the average total return of the portfolio and benchmark.

2. All earned attributes compound into the future at the average rate of total return of the portfolio and benchmark.

- *Notice*: Due to the first assumption, attributes are earned on a base that is defined by the benchmark return plus half of the active portfolio outperformance. Even if this base reflects half of the earned Allocation effect, Selection over this component will be labeled Selection. However, since only half of the earned Allocation effects will be present in the base, only half of the cross product will be labeled Selection. Therefore, the first assumption labels half of the cross product to the effect that occurs last, Selection.

- *Notice*: Due to the second assumption, attributes earned in prior periods are carried forward by the benchmark return plus half of the active portfolio outperformance. Even if this future return reflects half of the portfolio's active Selection return, the Allocation earned in prior periods will be compounded into the future by this return. However, since the Allocation is compounded by only half of the portfolio's total active Selection return, only half of the cross product will be labeled Allocation. Therefore, the second assumption labels half of the cross product to the effect that occurs first, Allocation.

As Cariño mentions, it is difficult to label one of the recursive methods as being more "correct." The Frongello method is widely accepted around the world and intuitive defenses can be found in Frongello (2002), Laker (2002), Cariño (2002), and Bonafede, Foresti and Matheos (2002).[4] I'm suspecting that the wide following of this method is due to the intuitive appeal of the assumptions outlined above. However, Cariño provides a very sound argument in defense of the Reverse Frongello Algorithm. Although, I suspect the Reversed Frongello method is used less often, assuming an analyst accepts a somewhat less popular (though still valid) opinion about the cross product term, this algorithm is valid. Finally, I would like to offer for consideration the Modified Frongello Algorithm. Unlike the Frongello Algorithm that assigns the cross product terms to the later effect, and unlike the Reverse algorithm that assigns the cross product term to the prior effect, the Modified Frongello Algorithm assigns half of the cross product term to the prior effect and half to the later effect. In conclusion, I agree with Mr. Cariño's observation and although one recursive method may have more intuitive appeal than another, perhaps the analyst should have discretion over which he or she chooses.

REFERENCES

Bonafede, Julia, Steven Foresti, and Peter Matheos, Ph.D., "A Multi-Period Linking Algorithm That Has Stood the Test of Time" *The Journal of Performance Measurement*, Fall 2002, pp. 15–26.

Cariño, Ph.D., David R., "Refinements in Multi-Period Attribution," *The Journal of Performance Measurement,* Summer 1999, pp. 45–53.

Frongello, A. "Linking Single Period Attribution Results," *The Journal of Performance Measurement,* Fall 2002, pp. 54–67.

Mirabelli, A. "The Structure and Visualization of Performance Attribution," *The Journal of Performance Measurement*, Winter 2000/2001, pp. 55–80.

ENDNOTES

[1] G_{tb} = Original attribute b in time period t,
F_{tb} = Adjusted attribute b in time period t,
R_t = Portfolio total return in time period t and
$\overline{R}_t$ = Benchmark total return in time period t.

Each illustrated algorithm is composed of two components. The first piece represents the portion of the current period's contribution to excess return due to active decisions in the current period. The second piece represents the current period's contribution to excess return due to the "echo" of past active decisions compounding in the present period at some current rate of return. Perhaps we could add some value by presenting these components separately.

	Portfolio	Benchmark	Diff	Allocation	Selection
Period 1	21.00%	11.00%	10.00%	6.00%	4.00%
Period 2	14.00%	9.00%	5.00%	2.00%	3.00%
Period 3	20.00%	12.00%	8.00%	1.00%	7.00%
Period 4	17.00%	10.00%	7.00%	5.00%	2.00%
Total	93.67%	49.06%	44.61%		

	Frongello		Reverse Frongello		Modified Frongello	
	Allocation	Selection	Allocation	Selection	Allocation	Selection
Period 1	6.00%	4.00%	6.00%	4.00%	6.00%	4.00%
Period 2	2.96%	3.99%	3.06%	3.89%	3.01%	3.94%
Period 3	2.45%	10.61%	3.02%	10.05%	2.74%	10.33%
Period 4	9.42%	5.17%	8.83%	5.76%	9.11%	5.48%
Total	20.83%	23.78%	20.91%	23.70%	20.86%	23.75%

[2] A four period example and comparison between the methods. In most cases, these methods will produce relatively similar results, especially during periods of strong correlation between portfolio and benchmark returns (see table above).

[3] Jose Menchero brought the Reverse Frongello Algorithm to my attention some time ago. We noticed that these results equal the results of the Frongello Algorithm when the periods are reversed. Unfortunately, I did not appreciate the intuitive rationale behind the Reverse method until I read David Cariño's most recent paper.

[4] The method described by Bonafede, Foresti and Matheos (2002) is mathematically equivalent to the Frongello Linking Algorithm in that it produces the same cumulative level results. However, for those interested in the single period supporting detail of the cumulative level results, I would suggest that those apply the single period scaling described in Frongello (2002) for the following reasons:

1. Non A-Causality and

2. Sincerity.

First, the mechanics of the linking described by Frongello (2002) ensures that there is *no a-causality* (Mirabelli 2000) in the scaled single period results. By this I mean that current period results are not dependent on future returns that have not occurred yet. The mechanics described by Bonafede, Foresti and Matheos (2002) require information about the benchmark rate of return between the current period and the end of the cumulative period. For example, June's contribution to the current year's attribution will not be known until the year is complete and the year's benchmark returns can be used to scale June's results. In contrast, the inputs and mechanics of the Frongello single period scaling requires no information about future returns. (*i.e.* June's contribution to the current year can be ascertained in June). Any future "echo" effect that occurs will not be quantifiable (attributable) until the future periods occur and the returns are known.

Second, to ensure the *sincerity* (Frongello 2002) of the attribution linking, the period's contribution to outperformance should be attributed to the period in which it occurred. For example, if a period contributes 10% to relative outperformance, then the attribution should attribute 10% of relative outperformance to that period. Because the mechanics described by Bonafede, Foresti and Matheos (2002) multiply the current attribute by the future benchmark return, the current period's contribution to excess return will not agree to the current period's contribution to attributed return.

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